ABACUS BY TOUCH

By Curtis L. McKee, B.S.

Chief, Communication Skills Department

Criss Cole Rehabilitation Center

State Commission for the Blind

Austin, Texas

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INTRODUCTION

Throughout history, blind people have been constantly frustrated by the lack of an effective and simple method of working arithmetic. Though many devices and methods of working arithmetic have been offered by the schools for the blind throughout the country, these devices were cumbersome and did not lend themselves to the ease and portability of the pencil and paper used by sighted students. Blind businessmen, students, housewives, etc., in all their activities, have constantly had to spend grueling hours working simple arithmetic problems, with a numberframe of some type, a slate and stylus, a brailler, or it was necessary to call upon their sighted friends for help. None of these methods have been really effective, and blind people have long voiced their desire for a simple and better way of working arithmetic.

Due to the extensive work done by Mr. T. V. Cranmer, Director, Division of Services for the Blind, Kentucky Department of Education, in developing the Cranmer abacus, a method has finally been perfected that allows the blind person to work his arithmetic problems as easily and accurately and with more speed than any of his sighted friends can work theirs with pencil and paper. The Cranmer abacus has removed the restrictions from a blind person handling all his arithmetic needs, and becoming completely independent in this area as he had done in other areas long ago. I do not feel that enough appreciation can be expressed for the work done by Mr. Cranmer for making this device available to those of us who are blind. We are indebted to him as we are to Louis Braille who many years ago gave us a practical and easy method of written communication with ourselves and others. Mr. Cranmer

has placed the whole field of arithmetic within easy reach of any blind person interested in it.

Also, many thanks and much recognition must be given to the two people who, primarily, made it possible for us to learn how to operate the Cranmer abacus. Mr. Fred L. Gissoni, author of USING THE CRANMER ABACUS FOR THE BLIND,[1] and Mr. Takashi Kojima, author of THE JAPANESE ABACUS, ITS USE AND THEORY,[2] have also played a very big part in freeing blind people from the "arithmetic chains" which had us bound for so many years. Both books are well written texts, and demonstrate that a tremendous effort and research went into their development. The Cranmer abacus gave us a tool with which to work, but without the precious knowledge of knowing how to utilize it, we would not have had the good fortune of now having an easy, workable method of arithmetic computation. Thanks to the effort of Mr. Gissoni and Mr. Kojima, we now have that information.

I first heard of Mr. Gissoni's book, USING THE CRANMER ABACUS FOR THE BLIND,[3] the Fall of 1963 while in training at the Kansas Rehabilitation Center for the Blind, Topeka, Kansas. I was in training for the position I was soon to hold as Home Teacher for the Blind, working for the Division of Services for the Blind and Visually Impaired, Kansas State Department of Social Welfare. There was a great deal of excitement in the center when Mr. Gissoni's book was received, and a number of us set about immediately and eagerly to determine if the Cranmer abacus really served the purpose

[1] Gissoni, Fred L., Using the Cranmer Abacus for the Blind, American Printing House for the Blind, Louisville, Kentucky.

[2] Kojima, Takashi, The Japanese Abacus, Its Use and Theory, Charles E. Tuttle Company, Rutland, Vermont; Tokyo, Japan, ©1954. Braille Edition-Braille Association of Kansas, Wichita, Kansas.

[3] Gissoni, _loc_. _cit_.

for which it was intended. My excitement and interest was more intense because I have always been a "math bug" and had always been frustrated by the methods previously available. I soon discovered that the Cranmer abacus did provide the means of allowing a blind person to work his arithmetic computations with ease, accuracy, and independence, and I became more excited about the abacus as my speed and proficiency increased. I immediately began introducing the abacus to my clients, and for eight years while serving as a Home Teacher for the Blind, I taught the use of the Cranmer abacus with the help of the books by Mr. Gissoni and Mr. Kojima.

During those years as Home Teacher and later as Instructor for the Blind, Kansas Rehabilitation Center for the Blind, Topeka, Kansas, as well as in my present position of Chief, Communications Department, Criss Cole Rehabilitation Center, Texas Commission for the Blind, Austin, Texas, I have been aware of the need for a more comprehensive approach to teaching the abacus to blind people. Throughout this time, I have experimented with various techniques in the actual mechanical aspect of the abacus, trying to develop a good, effective way of manipulating the beads with the most accuracy, speed, and without vision. I have continued experimenting with different techniques and feel that I have developed a uniform method of approach in teaching the theory, as well as the technique, of manipulation of the Cranmer abacus to blind people.

In this text, I have tried to develop a teaching technique along with the necessary drill exercises to allow a new teacher to learn to use the abacus as well as to teach it. Since most of my work has been with blind adults, many of whom were newly blinded, it has been necessary to develop a systematic approach and method of operating the abacus for both the congenitally and adventitiously blinded adults. I believe that my method takes

into consideration all the factors surrounding the total rehabilitation of a blind adult, and I have purposely gone into great details throughout the following pages to make this method clear for any teacher who would want to use it.

With the sudden overwhelming increase in new rehabilitation centers for the blind being set up throughout the country and with the limited supply of well-trained people who have had experience working with and teaching blind people, it has been necessary to hire many teachers from the public schools who have had no experience in either teaching the abacus or teaching blind people. Since no comprehensive instructional material is available to these new teachers who for the first time are faced with teaching a subject of which they know little and working with blind people with whom they have had no previous experience, I feel that this text will meet the total need for understanding the theory and technique of manipulating the abacus, as well as providing these new teachers a better understanding of how to approach teaching blind people in a more effective way. Besides providing a systematic approach of teaching the theory and manipulative skills in operating the abacus, I have also emphasized many factors about blindness which any teacher of the abacus working with a blind person needs to know. As previously mentioned, many of these new teachers have had little or no experience in teaching the use of the abacus, so I have tried to lay-out a lesson plan that will offer a systematic approach as well as providing a great deal of discussion as to how the material should be presented. This is certainly not to say that my method is "the" method, since there are many good abacus teachers using various methods throughout the country, but I do feel that it is one method which can better facilitate the teaching of the abacus to the blind.

Needless to say, future revisions will have to be made incorporating new knowledge gained through new experiences. I hope that in these revisions I will be able to utilize the experiences and knowledge gained by teachers throughout the country engaged in teaching the abacus to blinded individuals. In fact, I would like to suggest that sometime in the future a committee be formed of teachers involved in teaching the abacus with the purpose to develop the best technique and approach possible.

Perhaps it is time that the public schools involved in teaching blind students and schools for the blind take a long, hard look at the methods they are presently using in teaching arithmetic and consider the results emerging from the rehabilitation centers throughout the country. Possibly it is now time to initiate the teaching of the use of the abacus in these schools, either as a primary tool for working arithmetic, or combining its use with the present devices which are being used. I personally believe the abacus to be superior to any of the other methods used in teaching arithmetic to blind students, and it seems to me to be unfair to these students not to consider the abacus as a means of teaching arithmetic. Hopefully, as more knowledge is gained about the effectiveness of the abacus, more schools will turn to it as their primary device for arithmetic computation.

This text could easily be adapted for use by public schools serving blind students and schools for the blind. Perhaps some revision would be necessary as far as the technique of manipulation, as well as the approach in presenting the material, is concerned, since the situation would be so extremely different than teaching blind adults who already have knowledge of numbers and arithmetic.

With the speed, accuracy, portability, economy, and lack of strenuous mental calculation offered by the abacus, I do not see how we can avoid using and teaching it to blind students at all ages and levels of education.

Another area for consideration of use of the abacus which has previously been overlooked is the special education classes in public schools serving slow learners. Due to the simplicity of its operation, the abacus would perhaps be an ideal tool for these students.

HISTORY OF THE ABACUS

The abacus, though extremely popular in the Orient today, was probably first used by the ancient Romans to carry on their business transactions. According to Mr. Kojima,[1] the abacus was probably first used in ancient Rome and took the form of a dust-covered table on which figures were drawn and erased with the finger or a stylus. The English word abacus is derived from the Greek word abax which means a reckoning table covered with dust. This type of abacus gave way to a table on which lines were drawn and numbers were represented by discs which were placed at various positions on the lines. This particular type of abacus received widespread popularity in Europe until around the first part of the 17th century. As writing material became more available, however, the abacus was abandoned by the Europeans, who then began to depend solely on graphic arithmetic computation.

In Rome, there was yet a third type of abacus used at one time. It was a table with grooved lines, and counters were rolled up and down the grooves to represent numbers. There is evidence that the grooved-line abacus was used in China as early as the 6th century, and it probably was brought there by merchants who transacted business affairs with the Romans. In China, the grooved-line abacus gave way to a fourth type of abacus which consisted of a frame in which rods were set parallel to each other and beads were moved up and down on these rods to represent numbers.

This rod or bead abacus was probably developed about the 12th century

[1] Kojima, op. cit., p. 2

and became more widespread in its popularity in the 14th century. In the 17th century the Japanese adopted the abacus and it received widespread popularity there in that century. It has undergone many changes and improvements in both design and technique of operation. The Chinese continue to use an abacus with seven beads on each rod--two above the bar and five below. The Japanese, on the other hand, have simplified their abacus to five beads on each rod--one above the bar and four below. In Japan the abacus is known as a soroban and it is used in almost all business transactions. Its use is taught in all schools, including those serving blind students.

The Cranmer abacus which is adapted for blind people and produced by The American Printing House for the Blind, Louisville, Kentucky, is a form of the Japanese soroban in that it contains only five beads to each rod. Mr. Cranmer designed it in such a way that it allows the blind operator to move the beads easily, but with enough resistance that they are not accidentally moved. This also facilitates feeling the beads without accidentally moving them.

The abacus has been proven to be much superior to the electric calculating machines used in this country today, and the popularity it enjoys in the Orient today attests to its reliability. As one of my clients punned, "The abacus is certainly a device you can count on." Though the abacus has had a long successful history in the Orient, early Europe and ancient Rome, it appears that its history in this country is going to be made in the future. It certainly promises a great hope for the future in serving blind people as their method of meeting their arithmetic needs.

"Aba cuses, but I don't mind, she gives me the right answer every time."

PURPOSE OF THE ABACUS

The primary purpose in developing a course in the use of the abacus is to offer the opportunity of restoration of arithmetic skills to the newly blinded individual and provide a better way of working arithmetic for the congenitally blinded person or to those who were blinded early in life. This course lends itself well to a rehabilitation center setting since restoration of arithmetic skills is certainly a part of the blind person's total rehabilitation. It would be impossible to consider an individual rehabilitated if this need is neglected. Too, it can lend itself as well to schools where arithmetic is an integral part of a student's education.

The abacus offers a simple, easy to operate, practical, fast and economical way to meet all daily arithmetic needs.

To the housewife, for example, it offers a simple means of keeping tabs on the family budget. Checks can be totaled easily showing bank balances and done with a minimum of effort. Too, the abacus can be used by the housewife as she does her grocery shopping. This is especially true for those of us who must watch our budget. Also, with the abacus, it is possible to know to the cent just what your total purchase is going to cost before you get to the check-out counter. This will eliminate those embarrassing moments when you find that you have more groceries than money. Income tax forms are now available in braille, and for the person who wants to make out his own income tax return, the abacus offers a quick, efficient and accurate means for doing just that. Another common and helpful use made of the abacus is to jot down (set) a telephone number until it can be

recorded permanently or discarded when it is no longer needed. It is easy to remember a phone number which you have just gotten from the telephone operator, but too often when you call the number and get a busy signal, you find that the number has already been forgotten. For the card and domino player, the abacus offers an easy way to keep scores without pulling out the old slate and stylus or trying to remember all the scores in your head. This latter method often leads to some rather "hot" disagreements and often terminates games early.

Businessmen are able to use the abacus in carrying out their daily financial transactions. Vending stand operators find it makes tallying a sale simple and accurate, as well as providing an easy way of keeping records up to date. For the more venturous homeowners who want to know how much their home is "really" costing over the next thirty years, the abacus offers a quick and easy way of determining this. Also, if knowing what the house is really costing isn't enough, he can then determine how much the family car is "really" costing. It offers the blind vacationer who sometimes feels that he is not contributing as much to the trip as he would like, since he is unable to share the driving responsibilities, an opportunity to take a more active part by keeping all the records of food, motel costs, recreation, gasoline, etc. It is also a good way to check that gas mileage to see if the salesman was selling you a bill-of-goods when you bought the car. For those who play the stockmarket, the abacus can quickly determine for you just how much money you have suddenly lost or perhaps gained. It enables the farmer to keep records on profits and losses as far as crop production is concerned, as well as helping him determine depreciation of his expensive farm equipment.

There are endless ways the abacus can benefit a blind person throughout his life, and it is important to discuss with a student the many possible ways in which it might serve him particularly. He will approach the task of learning to operate the abacus with more vigor and enthusiasm if he really sees a need for it.

The abacus is really quite "habit forming" and students get addicted to it quickly. They find themselves constantly taking it from either their pocket or purse, where it should be carried at all times, and working problems which were brought to their attention or just making up problems and working them.

INTRODUCING THE ABACUS

Now that the student is familiar with the history and development of the abacus and understands the ways in which the abacus can serve him, it is time to introduce the abacus itself.

The teacher should place the abacus in front of the student in the proper position so that the lower part of the abacus, which is the section that contains four beads, is next to him. In order to avoid developing bad posture habits and unnecessary fatigue, the teacher should emphasize the importance of sitting straight, with the hips back against the chair, and both feet on the floor. The body should be sitting square to the table. Encourage the student to explore the abacus with both hands, and while he is doing so, discuss with him the various parts of the abacus. It is extremely important to keep in mind that some congenitally blind students do not have the proper concepts necessary to understand the simplest descriptions. Terms such as up and down, top and bottom, front and back, etc., may have almost no meaning or a completely different meaning than the meaning the teacher has of it. Make sure that all terms being used are understood by all the students.

The abacus is rectangular in shape and should be operated in the position in which it is placed in front of the student. Along the bottom edge, that is the part nearest to the student's body, is a series of dots and markers which can be felt, and this identifies it as the lower edge. Explain to the student that these markers will be discussed later when they are brought into use. The abacus consists of a frame containing thirteen rods which extend from the top to the bottom of the frame. The rods

are arranged in a parallel position, and each rod contains a total of five beads which slide up and down it. The bar, which runs from left to right across the rods, divides the beads into two sections. The upper section contains one bead on each rod. Below the bar, in the lower section, there are four beads on each rod. It is necessary to give some students more time to explore the abacus in order to develop a good mental picture of it. If there appears to be confusion on the part of the student, then physically place the student's hands on the different parts of the abacus as it is being discussed.

The terms used in operating the abacus are different than those used in arithmetic computation previously used by the student in print or braille and a brief discussion of these new terms will eliminate confusion. The abacus is made up of a frame, rods, beads, and bar as explained earlier. Numbers are "set" (written) by moving the beads to the bar. This gives the beads value. Numbers are "cleared" (erased) by moving the beads away from the bar, thus removing their value. Each bead below the bar has a value of one when set, and each bead above the bar has a value of five when set.

Since all numbers formed on the abacus, with the exception of the number five, involve one or more beads below the bar, it is important to first learn to manipulate the lower four beads. The five-bead will be used later, and its concept is easily understood once the theory of the lower four beads is mastered.

Not only is it important to learn proper bead manipulation, but it is equally important that the hands be used in the proper way. The elbow should be kept close to the body with the wrist resting comfortably on the table. The forefinger of the right hand is always used to manipulate

the beads, and should always be kept in a position that allows the finger to run parallel to the rods on which the manipulation is being done. Never allow the student's right hand to come in from the end of the abacus or at a slant. With the wrist resting comfortably on the table, the forefinger is in a natural position for moving the beads up and down the rods. Rest the left hand on the abacus, with the forefinger resting lightly on the third rod from the right, just below the bar. This finger helps to keep the proper position and serves as a guide. Later, the left hand will serve to meet a variety of needs, but should begin manipulating beads as soon as possible. Often students with good hand coordination are able to manipulate the upper bead with the left hand while manipulating the lower beads with the right hand from the very beginning. The abacus should always be operated sitting flat on the table. As the forefinger of the right hand manipulates the beads, it should always come to rest on the open rod and never should be brought to rest on a bead. This prevents any accidental moving of the beads.

READINESS

Now that the student is familiar with the abacus and the terms to be used in operating it, it is a good time to start the readiness program. Accuracy is the most important factor in operating the abacus successfully; therefore, it is necessary to help the student develop a good understanding of the three basic principles of setting, reading, and clearing numbers. This does not include any of the procedures involved in addition, subtraction, multiplication, or division. This is a familiarization period during which time the student trains his fingers to operate the abacus mechanically with ease and accuracy. The length of the readiness part of the course depends on how rapidly the students are able to grasp the theory of setting and reading numbers and to a great extent, on how well they are able to use their hands.

In order to get the "feel" of the abacus, it is best to start by setting simple numbers from one to four, reading them, and then clearing them. Since the right end of the abacus is used in addition and subtraction, that is a good place to begin. With the hand in the proper position, set one, that is, push up one bead, then rest the finger on the rod below. Now feel the one which has been set by gently moving the tip of the finger up and over the bead until it touches the bar, and back down again. It is important to think of this as the number one, and not just a single bead.

Set the second bead on the same rod by simply moving up another bead as far as it will go. Rest the finger on the rod below. Again, gently move the tip of the finger up and over the two beads until it touches the bar, and move it down again to the open rod below. Mentally think of this

as the number two and not just two beads. Note, it is very important that the student develop a good mental picture of each number as he forms it, and this is sometimes difficult, since the numbers on the abacus "look" so different from the numbers he has always read before. Set a third bead following the same instructions and procedure as before, i.e., set bead, rest finger below, gently feel the number, then think of the number which the beads represent. Set the fourth bead, following the same procedure as before.

Now that the numbers one through four have been set and read, gently move the finger up to the bar and clear all four beads with one downward motion. Rest the finger on the rod immediately below the bar. Repeat this exercise a number of times until the student is moving the beads up and down the rod with ease. Remember, the finger should be parallel to the rod on which the beads are being manipulated.

Change the procedure a little at this point by setting one, and after feeling it, clear it. Rest the finger on the rod immediately below the bar. Now, set two, feel it and clear. Set three and four following the same procedure. After the student has done this a number of times and is able to do it accurately, ask him to move to the next rod to the left and continue setting numbers from one to four. Emphasize that the beads on all rods have the same value, and they are all manipulated in the same way. The forefinger, left hand, should remain on the third rod. Remind the student that the empty rod to the right of where he is working is a zero and not just an empty rod.

A new procedure to introduce at this point is to ask the student to set a one on the second rod, move the finger up to the bar and to the right, then set a one on the first rod. This gives him the number eleven.

Note, numbers on the abacus are always set and cleared from left to right. Now clear the eleven by moving to the left, clearing the one on the second rod before clearing the one on the first rod. Let the finger come to rest below the bar on the first rod.

The numbers 11, 22, 33, 44, 12, 13, 23, etc. are good combinations to set, read, and clear. Then include the third rod by setting numbers with three digits. When the students are able to remember and set three digit numbers, then move on to four and five digit numbers.

Now that the student is able to accurately set, read, and clear numbers from one to four, introduce the five-bead above the bar. To make numbers six, seven, eight and nine, you simply add the lower four beads to the five-bead one at a time. When each number is formed, be sure the student has plenty of time to feel the number before clearing it. In setting or clearing numbers where the five-bead is involved, the five-bead is always moved first. Set the numbers six through nine using the same rod until the students are able to set, read and clear them accurately and without difficulty. Then set numbers of more than one digit using numbers from one to nine inclusively. Start with setting a few two digit numbers such as 19, 28, 37, 46, etc. Then expand this to three numbers.

If the student is able to handle these numbers well, then let each student suggest numbers ranging from one to six or seven digits. Addresses, phone numbers, years of birth, and all types of numbers will serve to give practice in this area. Students appreciate being involved and making decisions, and this should be encouraged. When the abacus is being worked with ease and confidence, then it is time to end the readiness program and initiate the skill exercises. This is a very good time to review with

students the importance of good posture, such as keeping the elbow in close to the body and the proper finger manipulation in operating the abacus. Also, be sure to stress that each student should feel that he is not being pressured and that he is not in competition with any of the other students. Emphasis is placed on each person restoring to himself his previous arithmetic skills.

SKILL EXERCISES

Skill exercises are drills which, through constant practice, train the fingers to accurately manipulate the abacus. It is impossible to add numbers in arithmetic problems, without first knowing how to add a number to itself, i.e., 1 plus 1 plus 1, etc.; 2 plus 2 plus 2 plus 2; and on up to nine. Drills are sometimes considered tedious, but it is important that the abacus operator knows how to add numbers to themselves before going on to actual problems. It is through these skill exercises that the students learn the "complementary" numbers which are used in the operation of the abacus. Complementary numbers are those numbers which are added together to equal either five or ten. For instance, 1-4, 2-3, are complementary numbers equaling five and 1-9, 2-8, 3-7, 4-6, 5-5 are the complementary numbers equaling ten. Complementary numbers should be memorized and repeated by the abacus operator over and over.

The following rules or secrets are rotely taught by some abacus teachers. They are taught in all the schools in Japan, and mastery of them is stressed as mastery of the multiplication tables are in our schools. I find that through practicing the drill exercises, it is easy to understand the theory behind the rules, and it is not necessary to rotely memorize these rules. Perhaps the theory of teaching these rules would be more applicable in a grade school setting than in a rehabilitation center.

The rules, which I will use in this text, stress the importance of setting your one to the left before clearing the complement and setting the five bead before clearing the lower beads. Americans are in the habit of paying someone money and getting back change which is the same concept.

Complementary Numbers in Addition:

To add	Set	Clear	Set
1	5	4	
1	1 left	9	
2	5	3	
2	1 left	8	
3	5	2	
3	1 left	7	
4	5	1	
4	1 left	6	
5	1 left	5	
6	1 left	4	
6	1 left	5	1
7	1 left	3	
7	1 left	5	2
8	1 left	2	
8	1 left	5	3
9	1 left	1	
9	1 left	5	4

It has been my experience that students develop their own technique and system of setting numbers and clearing complements. Since the abacus is a mechancial device, it is important that the students follow a systematic approach in its manipulation.

The student who has first learned to follow the procedure of setting 1 left before clearing the complement right may, after gaining confidence and skill on the abacus, prefer to reverse this procedure and subtract the complement before adding the 1 left. Since this reversal would possibly increase his speed and accuracy, do not discourage him from doing so.

The following exercises should be performed on the first two rods, on the right end of the abacus:

ONES: On the first rod, the unit rod, set the number 1 and rest the finger below. Add 1, remembering to always rest the finger on the rod below. Add 1 and add 1. Now that we have added 1 to itself to reach the sum of 4, we are ready for the first rule or complement. To add 1 to 4, you set 5 and clear 4, i.e., the difference between 1 and 5. So the complement of 1 is 4. In setting the 5 and clearing the 4, simply allow the finger to make a single downward motion setting the 5 and clearing the 4 with one movement. Now, add 1 to 5, 1 to 6, 1 to 7, 1 to 8, and then clear the abacus and repeat the procedure. Remember to clear the five-bead before clearing the lower four beads.

Students should be encouraged to verbalize outloud the procedure for adding 1 to 4--"set 5 clear 4". If you can not set 1, you set a 5 and subtract the difference which is 4. If the student understands that he has followed this procedure in making change with his money all his life, he can easily see how and why it is done on the abacus. The idea of 1 cent and 4 cents equaling 5 cents is meaningful. If you owe 1 cent and have only a nickle with which to pay, you get 4 cents change. The abacus is worked only with pennies, nickles and dimes.

When the student is able to add 1 to itself until the number 9 is formed easily and accurately, then it is time for the second rule which is to add 1 to 9, you set 1 left and clear the 9, i.e., 9 being the difference between 1 and the 10 which was set to the left. This leaves the abacus with one bead on the second rod and the first rod cleared. This is exactly how electric adding machines work as well as the odometer of a car. Once 9 has been reached on a column, you have to set 1 to the left and clear the 9 from the column to the right. Thus, the second complement of 1 is 9. Ask the student to repeat 1-4, 4-1, and 1-9, 9-1 over and over. Again,

you have owed a penny and paid with a dime, so it is necessary to subtract the nine cents change.

Clear the abacus and repeat the procedure of adding ones until 10 is reached. After repeating this a number of times, continue to add ones above the number 10 until the number 19 is reached. To add 1 to 19, you simply set one to the left and clear the 9, leaving a total of 20 on the abacus. Continue adding ones to 29, and to add one to 29, set one left and clear the 9. Now the abacus reads 30. Explain that all numbers set to the left on the second rod are tens.

It is a good policy at this point to let the student add 1 to itself on the second rod to see that it is done exactly as it is on the first rod. Therefore, when he reaches the number 49 in his addition, he will find it necessary to set 5 and clear 4 on the second rod before clearing the 9 on the first rod.

<u>TWOS</u>: Set 2, add 2, and rest finger below. To add 2 to 4, it is necessary to set 5 and clear 3, i.e., 3 being the difference between the 2 we wanted to add and the 5 that we actually added. Thus, the complement of 2 is 3. Add 2, making a total of eight. Clear and repeat adding twos until 8 is reached. When you reach 4 each time, verbalize that it is necessary to set 5 and clear 3, the complement of 2. Now, to add 2 to 8, set 1 left and clear 8, which is the difference between the 10 you set to the left and the 2 you were adding. Thus, the second complement of 2 is 8. Repeat this exercise allowing the student to add until he reaches the number 52.

<u>THREES</u>: Set 3, add 3 by setting 5 and clearing 2. Thus, the complement of 3 is 2. Add 3 to the 6 formed on the abacus, clear, then repeat adding threes. To add 3 to 9, you set 1 left and clear 7, the second

complement of 3. Be sure to clear the five-bead before clearing the two lower beads. Continue adding threes until the number 51 is reached.

FOURS: Set 4, add 4 by setting 5 and clearing 1. Thus, the complement of 4 is 1. Clear the abacus and repeat adding fours. To add 4 to 8, you set 1 left and clear 6, the second complement of 4. Continue adding fours until 96 is reached. At that point, it is necessary to utilize the third rod on the abacus to add 4 to 96. Since we can not add 1 to the 9 set on the second rod, we set 1 to the left, the third rod, and clear the 9. That procedure has added 10 to the 90 which was the previous number on the second rod, and now clear the 6 from the first rod, since it is the complement of the 4 we were originally adding. Clear, set 96 a number of times, adding 4 to it. Point out that the one-zero-zero formed on the abacus is the number 100 and that the third rod is the hundreds rod. Rather than adding 100 and clearing 90, however, as in the case of 96 plus 4, use the terms of setting 1 left and clearing the complement 9. This is the procedure to follow regardless of which rod is being used.

FIVES: There is no need to practice setting fives, since you simply add 1 to the left and clear the 5. Thus, the complement of 5 is 5.

SIXES: Sixes give students a little more trouble than the other numbers because it is necessary to learn a second procedure in order to add sixes. It is necessary to learn to subtract fours. Set 6, the five-bead first, and to add six, we set 1 left, subtracting or clearing 4. To clear 4, however, it is necessary to clear 5 and set 1 on the first rod.

A good exercise to practice here is to set numbers four through nine, subtracting four each time and clearing the abacus. For instance, subtract 4 from 9 and clear. Set 8, subtract 4 and clear, and so forth.

This particular skill exercise eliminates the difficulty in adding sixes. Continue adding sixes until the number 102 has been reached.

SEVENS: Set 7. To add 7, set 1 left, clear 3. Again, 3 being the complement of 7. To clear 3, it is necessary to clear 5 and set 2. Therefore, before continuing to add sevens, practice the skill exercise of subtracting 3 from 9, from 8, from 7, etc., until you subtract 3 from 3. Once the student has mastered subtracting threes, then continue with adding sevens until the number 105 is reached.

EIGHTS: Set 8. To add 8, set 1 left, clear 2. Thus, the complement of 8 is 2. Again allow the student to practice subtracting 2 from the numbers 2 through 9. Once this has been practiced and is accomplished with ease and accuracy, continue adding eights until the number 104 has been reached.

NINES: Nines are easy to add since you simply set 1 to the left and clear 1. Add 9 to itself until the number 108 has been reached.

This completes the skill exercises, but it is very important that the abacus operator learn these skills well and is able to perform them with ease and accuracy. They should be practiced constantly, even after the operator has become skilled with the abacus. It is a good way to "warm-up" at the beginning of class periods. Review with the student the complementary numbers, starting with 1-9 and going up to the number that he is presently working on. These should be repeated verbally, and the student should learn to give an automatic response when a complement is stated.

COMPLEMENTS: 1-4, 4-1, 2-3, 3-2, 1-9, 9-1, 2-8, 8-2, 3-7, 7-3, 4-6, 6-4, 5-5.

To prevent these exercises from becoming boring, there are many variations such as starting with a different number than you are adding, i.e., set the number 2, then add threes until the number 53 is reached. Set 3 and add fours until the number 103 is reached, etc. Also, start adding with 1 plus 2 plus 3, etc. Numbers 1 through 9 added together equal 45.

ADDITION

One-Digit Numbers

Now that the student has learned to add numbers 1 through 9 to themselves accurately, let him work the following problems. I have tried to design these problems in a way that all numbers are used many times. These problems can be recorded and the student can work them from a tape recorder. A set of dominoes can be used and will serve two very good purposes--giving the student experience in using his hands in reading the dominoes, and giving him numbers which can be added. The number on each end of the domino can be added separately or the ends totaled, and the one number added.

Exercise I:

1. 2 & 6 & 5 & 1 & 8 & 7 & 3 & 2 & 9 & 4 = 47
2. 4 & 3 & 1 & 6 & 6 & 2 & 6 & 2 & 3 & 1 = 34
3. 7 & 1 & 5 & 9 & 8 & 2 & 8 & 7 & 6 & 9 = 62
4. 3 & 6 & 2 & 1 & 2 & 1 & 3 & 6 & 4 & 6 = 34
5. 7 & 2 & 8 & 9 & 7 & 2 & 5 & 6 & 1 & 8 = 55
6. 8 & 1 & 6 & 2 & 5 & 3 & 4 & 6 & 9 & 4 = 48
7. 3 & 1 & 8 & 9 & 3 & 6 & 5 & 6 & 9 & 7 = 57
8. 1 & 9 & 4 & 7 & 2 & 1 & 4 & 8 & 9 & 6 & 6 = 57
9. 2 & 7 & 3 & 5 & 4 & 2 & 6 & 8 & 3 & 1 & 7 = 48
10. 9 & 8 & 2 & 5 & 7 & 3 & 1 & 4 & 9 & 7 & 2 & 8 & 3 = 68

Exercise II:

1. 4 & 1 & 8 & 2 & 6 & 5 & 3 & 7 & 4 & 9 & 3 & 4 & 6 = 62
2. 7 & 1 & 8 & 6 & 3 & 2 & 5 & 9 & 2 & 6 & 7 & 4 = 60
3. 5 & 7 & 3 & 2 & 1 & 9 & 4 & 5 & 2 & 8 & 6 & 7 & 5 = 64
4. 2 & 3 & 6 & 7 & 8 & 4 & 9 & 6 & 5 & 8 & 2 = 60
5. 9 & 6 & 5 & 7 & 3 & 8 & 8 & 6 & 3 & 4 & 7 & 3 & 1 = 70
6. 8 & 5 & 6 & 3 & 2 & 1 = 25
7. 1 & 4 & 6 & 7 & 9 & 3 & 4 & 7 & 2 & 8 & 6 = 57
8. 2 & 5 & 8 & 7 & 2 & 6 & 5 & 6 & 9 & 1 = 51
9. 3 & 8 & 3 & 7 & 6 & 2 & 5 & 4 & 9 & 7 = 54
10. 6 & 3 & 2 & 3 & 4 & 5 & 1 & 6 & 4 & 3 = 37

Two-Digit Numbers

As in the case of one-digit numbers, there are skill exercises which help in learning to add two-digit numbers. A two-digit number is actually made up of two one-digit numbers placed side by side, and it is important to stress the procedure of always setting the numbers, as well as clearing them from left to right.

Skill Exercises: Set 11, add 11, add 11, until 99 is reached. To add 11 to 99, you set 1 on the third rod, clear 9 immediately to the right, and since we can not add 1 to the 9 on the first rod, it is necessary to set 1 left, on the second rod, and clear 9 to the right. This leaves the sum of 110. Repeat this procedure with 11, 22, 33, 44, 66, 77, 88, 99. Add each number ten times. This way the student can check his answer for accuracy when practicing out of class.

Continue with skill exercises by adding 12 to itself ten times, then 23, 34, 56, 67, 78, 89. By repeating the same number and adding it to itself, the student can give all his attention to the actual manipulation of the abacus and not have to spend time trying to remember the numbers which are being added.

The following problems have been designed to offer a wide variety of experience in using all the numbers.

Exercise I:

1. 23 & 49 & 51 & 12 & 34 & 85 & 28 & 96 & 67 & 74 = 519
2. 34 & 63 & 21 & 16 & 26 & 12 & 36 & 62 & 43 & 61 = 374
3. 87 & 12 & 68 & 29 & 57 & 32 & 45 & 60 & 91 & 18 = 499
4. 36 & 12 & 81 & 94 & 34 & 62 & 54 & 69 & 96 & 78 = 616
5. 17 & 93 & 46 & 77 & 25 & 19 & 47 & 89 & 99 & 61 = 573
6. 22 & 35 & 88 & 37 & 72 & 66 & 55 & 46 & 99 & 71 = 591
7. 65 & 36 & 26 & 50 & 43 & 54 & 12 & 61 & 44 & 15 = 406
8. 99 & 11 & 23 & 75 & 18 & 46 & 52 & 81 & 66 & 73 = 544
9. 51 & 56 & 44 & 53 & 16 & 62 & 21 & 50 & 45 & 34 = 432
10. 79 & 25 & 63 & 91 & 12 & 87 & 34 & 58 & 67 & 39 = 555

Exercise II:

1. 23 & 44 & 18 & 10 & 49 & 56 & 38 & 15 & 26 = 279
2. 75 & 12 & 38 & 26 & 63 & 81 & 19 & 27 & 10 & 41 = 392
3. 14 & 23 & 59 & 13 & 31 & 45 & 26 & 37 & 88 & 64 = 400
4. 14 & 53 & 12 & 72 & 37 & 29 & 81 & 68 & 94 & 27 = 487
5. 42 & 10 & 13 & 36 & 22 & 11 & 45 & 29 & 30 & 16 & 55 = 309
6. 24 & 19 & 35 & 72 & 67 & 39 & 15 & 22 & 12 & 93 & 40 & 57 & 18 & 10 & 61 & 29 = 613
7. 97 & 12 & 33 & 27 & 36 & 44 & 18 & 49 & 67 & 25 & 11 = 419
8. 27 & 18 & 39 & 46 & 38 & 51 & 19 & 73 & 64 & 41 = 416
9. 22 & 53 & 60 & 52 & 41 & 33 & 62 & 36 & 54 & 43 = 456
10. 43 & 64 & 54 & 42 & 55 & 63 & 30 & 21 & 40 & 31 = 443

Three or More Digits

There really is no difference in adding numbers of two digits and those containing more than two digits. The basic thing to emphasize is that numbers are always set and cleared from left to right. This is in keeping with the way that numbers are spoken, such 123, 234, etc.

The dots and markers which are found on the bar as well as the lower edge of the abacus can be used to help the student count over the appropriate number of rods that will be needed to set a number. Such as adding 12,345 to itself ten times, it is easy to use the rod-markers along the lower edge of the abacus to quickly find the fifth rod each time you start the number over.

In addition to the following problems which will serve to give plenty of exercise, the student can also add 123 to itself ten times, then add again including one additional number each time until he is adding 123,456,789 to itself ten times.

Exercise I: Three-Digit

1. 146 & 378 & 225 & 117 & 439 & 665 & 712 & 918 & 488 & 321 = 4409
2. 714 & 113 & 218 & 456 & 921 & 118 & 655 & 124 & 388 & 966 = 4673
3. 403 & 760 & 953 & 719 & 100 & 302 & 855 & 327 & 219 & 307 = 4945
4. 327 & 145 & 209 & 644 & 458 & 882 & 919 & 199 & 616 & 112 = 4511
5. 548 & 717 & 136 & 218 & 304 & 559 & 907 & 155 & 213 = 3757
6. 246 & 177 & 918 & 309 & 605 & 727 & 437 & 618 & 128 & 309 = 4474
7. 123 & 246 & 455 & 701 & 987 & 255 & 107 & 615 & 224 & 418 = 4131
8. 716 & 445 & 161 & 545 & 800 & 376 & 124 & 333 & 132 & 315 = 3947
9. 434 & 216 & 707 & 155 & 689 & 262 & 786 & 368 & 417 & 646 = 4680
10. 234 & 415 & 126 & 537 & 284 & 179 & 792 & 365 & 423 & 789 = 4144

Exercise II: Four-Digit

1. 1509 & 7122 & 3164 & 2305 & 1436 & 7118 & 5502 & 9099 & 7862 & 1102 = 46,219
2. 2007 & 3126 & 5044 & 2098 & 3217 & 1185 & 1492 & 1935 & 2177 = 22,281
3. 1655 & 3472 & 9554 & 3766 & 2518 & 5032 & 1678 & 1514 & 1302 = 30,491

4. 7112 & 3503 & 4799 & 8061 & 5327 & 4811 & 1245 & 3798 & 9055 = 47,711
5. 1777 & 1819 & 3660 & 2504 & 4381 & 9192 & 6103 & 4407 & 3259 = 37,102
6. 3748 & 1112 & 2464 & 4544 & 9177 & 6033 & 1788 & 5050 & 2347 = 36,263
7. 8999 & 2466 & 3132 & 7150 & 2038 & 1898 & 2918 & 1918 & 3518 = 34,037
8. 1935 & 2203 & 1717 & 2809 & 7567 & 4390 & 1788 & 7026 & 9591 = 39,026
9. 3645 & 9218 & 7032 & 1008 & 1930 & 7216 & 4529 & 1798 & 3211 = 39,587
10. 3476 & 2918 & 5077 & 3819 & 2008 & 3974 & 1441 & 2902 & 3755 = 29,370

Exercise III: Mixture

1. 1745 & 323 & 1478 & 25 & 4111 & 322 & 75 & 15,206 & 14 & 3782 = 27,081
2. 18 & 997 & 43 & 10,786 & 432 & 9 & 223 & 1788 & 63 & 127 & 2224 = 16,710
3. 519 & 147,625 & 318 & 27 & 455 & 6721 & 378 & 1497 & 399 & 12 & 418 = 158,369
4. 99 & 1 & 215 & 22,876 & 19,101 & 24 & 9 & 27 & 488 & 345,217 & 33 = 388,090
5. 246 & 13 & 35 & 2987 & 16,644 & 3 & 255 & 77 & 404 & 529 & 18,326 = 39,519
6. 886 & 329 & 157 & 424 & 350 & 22 & 15 & 1133 & 120,277 & 56 & 399 = 124,048
7. 4712 & 36 & 199 & 827 & 50 & 181 & 64 & 1009 & 651 & 32 & 299 & 8 = 8,068
8. 2347 & 12 & 319 & 55 & 30,728 & 119 & 88 & 254 & 1755 & 99 & 45 = 35,821
9. 17 & 451 & 133 & 85 & 509 & 17,822 & 355,072 & 14 & 9 & 326 & 578 = 375,016
10. 18 & 277 & 319 & 45 & 78,216 & 327 & 14 & 213 & 85,507 & 124 & 365 = 165,425

MONEY

In order to give students an opportunity to practice adding money, the following problems have been provided. Instead of trying to use a marker on the abacus for a decimal point, simply continue adding on the right end of the abacus letting the unit and tens rods represent the cents. Any number set left of these two rods would represent dollars.

Exercise:

$23.18	$ 1.99	$ .47	$ 3.65	$ 12.47
9.51	15.75	1.39	17.88	18.99
.38	7.89	2.79	104.92	.51
.72	.29	11.08	33.74	3.74
1.41	3.96	.21	18.00	1.18
3.77	14.25	5.00	5.44	7.79
2.19	2.47	3.99	.27	3.68
4.65	39.75	5.00	10.99	5.14
37.81	.23	28.14	14.26	317.84
8.22	.87	27.62	8.88	2.68
$91.84	$87.45	$85.69	$218.03	$374.02

$ 1.86	$ .98	$169.95	$ 20.15	$ 14.74
2.88	4.98	154.88	3.39	8.28
2.78	7.00	38.99	189.88	1.27
3.00	6.44	12.97	74.95	234.88
.47	8.88	8.99	259.39	159.95
.36	4.62	1.77	148.95	264.80
21.19	5.88	15.75	3.89	734.40
47.56	4.90	5.24	35.00	89.88
11.93	2.66	12.44	459.55	39.50
7.71	6.99	5.66	109.88	7.88
$99.74	$53.33	$426.64	$1,305.03	$1,555.58

SUBTRACTION

Students usually prefer to learn subtraction immediately following addition. However, some teachers prefer to teach subtraction after multiplication. For those teachers, this section could be skipped and returned to after the section on multiplication has been completed.

As I have pointed out numerous times, the five-bead is always set and cleared before setting or clearing the lower beads; we will continue to follow that method in this text.

Since we have been subtracting while adding, this part of the course is not difficult to master. We have already experienced subtracting twos, threes, and fours from numbers 2 to 9 inclusive, and this might be reviewed temporarily to refresh the student's memory of it.

<u>ONES</u>: Set the number 9, clear 1, clear 1, clear 1, clear 1, and to subtract 1 from 5, you clear 5 and set 4. Then it is no problem subtracting 1 from 4, 3, 2 and 1. Repeat this a number of times, then set the number 11, and begin subtracting ones. Eleven minus 1 leaves 10, and to subtract 1 from 10, you clear 1 left and set the complement 9. Continue subtracting. Repeat this starting with 11, 21, 31 and so forth. Not much time is needed for this exercise.

<u>TWOS</u>: Set 9, clear 2, clear 2, and to subtract 2 from the remaining 5, it is necessary to clear 5 and set 3, i.e., the complement of 2. Clear 2 from 3 and repeat the entire exercise. Set the number 12, and begin subtracting twos. Two from 12 leaves 10, and to subtract 2 from 10, you must clear 10 and set the complement 8. Continue subtracting. Repeat the exercise using 12, 22, 32, 42, and so on.

Continue these skill exercises with the numbers 3, 4, 6, 7, 8, and 9. First set 9 and subtract each of these numbers 3 through 9 as many times as possible. Then set a number larger than 10 and begin subtracting and continue until you can no longer subtract. Ninety-nine is a good number to start with in subtracting any of the numbers.

A good exercise is setting the number 45 and subtracting 1, minus 2, minus 3, and so forth, until you subtract 9, leaving zero.

The following subtraction problems will offer plenty of practice in subtracting one-digit numbers. These can be taped, and a student can work from the tapes. Since the answers are given, he can check his own accuracy.

Exercise I:

1. 55 - 8 - 1 - 6 - 5 - 2 - 7 - 9 - 8 - 2 = 7
2. 32 - 3 - 6 - 2 - 1 - 2 - 1 - 3 - 6 - 4 = 4
3. 68 - 3 - 8 - 2 - 7 - 9 - 4 - 1 - 3 - 6 - 8 - 6 = 11
4. 57 - 1 - 9 - 4 - 7 - 2 - 1 - 4 - 8 - 9 - 6 = 6
5. 47 - 4 - 9 - 2 - 3 - 7 - 8 - 1 - 5 - 2 = 6
6. 62 - 4 - 1 - 8 - 2 - 6 - 5 - 3 - 7 - 4 - 9 - 3 - 6 = 4
7. 64 - 5 - 7 - 6 - 8 - 2 - 5 - 4 - 9 - 1 - 2 - 2 = 13
8. 37 - 6 - 3 - 2 - 3 - 4 - 5 - 1 - 6 - 4 = 3
9. 51 - 1 - 2 - 5 - 9 - 6 - 2 - 7 - 8 = 11
10. 25 - 8 - 5 - 1 - 2 - 6 - 3 = 0

Exercise II:

1. 34 - 1 - 3 - 2 - 6 - 2 - 6 - 6 - 1 - 3 = 4
2. 48 - 2 - 7 - 3 - 5 - 4 - 2 - 6 - 8 - 3 - 1 = 7
3. 62 - 7 - 1 - 5 - 9 - 8 - 2 - 8 - 7 - 6 = 9
4. 48 - 4 - 9 - 6 - 4 - 3 - 5 - 2 - 6 - 1 = 8
5. 57 - 3 - 1 - 8 - 9 - 3 - 6 - 5 - 6 - 9 = 7
6. 60 - 4 - 7 - 6 - 2 - 9 - 5 - 2 - 3 - 6 - 8 - 1 = 7
7. 60 - 2 - 8 - 5 - 6 - 9 - 4 - 8 - 7 - 6 = 5
8. 54 - 3 - 8 - 3 - 7 - 6 - 2 - 5 - 4 - 9 = 7
9. 58 - 1 - 4 - 6 - 7 - 9 - 3 - 4 - 7 - 2 - 8 = 7
10. 70 - 1 - 3 - 7 - 4 - 3 - 6 - 8 - 8 - 3 - 7 - 5 - 6 - 2 = 7

Two-Digit Numbers

Subtracting two-digit numbers is not difficult, and it is only necessary to remember to work the abacus from left to right, keeping columns straight.

Set a number such as 99 and simply begin subtracting numbers like 22, 33, 44, 55, 66, 77, 88, and 99. In the case of 22, you can first set the number 220, i.e., 10 x 22, and with 33 set the number 330, etc., subtracting 22 or 33 until reaching 0. The student should be encouraged to use his imagination in developing problems to work.

In class, students have enjoyed making up problems such as adding only odd numbers, i.e., 1, 3, 5, 7, 9, 11, until they add the number 99, giving a sum of 2,500 and then subtracting back down the same way.

The following practice problems will give sufficient practice for learning to subtract accurately and with ease.

Exercise I:

1. 443 - 31 - 40 - 21 - 30 - 63 - 55 - 42 - 54 - 64 = 43
2. 279 - 23 - 44 - 18 - 10 - 56 - 49 - 38 - 15 - 13 = 13
3. 309 - 55 - 42 - 16 - 10 - 13 - 36 - 30 - 29 - 45 - 11 = 22
4. 416 - 41 - 64 - 73 - 19 - 51 - 38 - 46 - 39 - 18 = 27
5. 419 - 97 - 12 - 33 - 27 - 36 - 44 - 18 - 49 - 67 - 25 = 11
6. 519 - 74 - 67 - 96 - 28 - 85 - 34 - 12 - 51 - 49 = 23
7. 499 - 87 - 12 - 68 - 29 - 57 - 32 - 45 - 60 - 91 = 18
8. 406 - 65 - 36 - 26 - 50 - 43 - 54 - 12 - 61 - 44 = 15
9. 555 - 79 - 25 - 63 - 91 - 12 - 87 - 34 - 58 - 67 = 39
10. 573 - 61 - 99 - 89 - 47 - 19 - 25 - 77 - 46 - 93 = 17

Exercise II:

1. 374 - 61 - 43 - 62 - 12 - 26 - 16 - 21 - 63 - 34 = 36
2. 616 - 78 - 96 - 69 - 54 - 62 - 34 - 94 - 81 - 12 = 36
3. 591 - 71 - 99 - 46 - 55 - 66 - 72 - 37 - 88 - 35 = 22
4. 544 - 99 - 11 - 23 - 75 - 18 - 46 - 52 - 81 - 66 = 73
5. 432 - 51 - 56 - 44 - 53 - 16 - 62 - 21 - 50 - 45 = 34
6. 392 - 75 - 12 - 38 - 26 - 63 - 81 - 19 - 27 - 10 = 41
7. 400 - 64 - 88 - 37 - 26 - 45 - 31 - 13 - 59 - 23 = 14
8. 487 - 14 - 53 - 12 - 72 - 37 - 29 - 81 - 68 - 94 = 27

9. 613 - 24 - 19 - 35 - 72 - 67 - 39 - 17 - 93 - 52 - 57 - 57 = 81
10. 456 - 43 - 54 - 36 - 62 - 33 - 41 - 52 - 60 - 53 = 22

REMEDIAL ARITHMETIC

Since students attending rehabilitation centers vary in their educational level of achievement, it may be necessary before beginning problems in multiplication to review the multiplication tables. These tables are located in the back of this text and can be easily recorded for individual study. It is impossible to work multiplication problems without a working knowledge of the multiplication tables, so be sure the student knows these tables before advancing to the next section of this text.

MULTIPLICATION

Up to this point, we have used only the right end of the abacus in addition and subtraction. The left end of the abacus is always used for multiplication and division. To avoid confusion in terminology, it is much simpler to set problems in the order in which they are presented, i.e., 2 x 4 = 8, set the 2 before setting the 4. In this case, the 2 is set on the first rod to the left, and the 4 would be set to the right on the fourth rod. This leaves two vacant rods between the multiplicand and multiplier. The number set to the right, or "middle" of the abacus, is always the multiplier, since the number on the left will always be multiplied by this number. This procedure of always setting the numbers as they are presented, from left to right, and always calling the number to the right the multiplier eliminates the tedious job of trying to determine which number is the multiplier and which number is the multiplicand. Some operators of the abacus set their multiplicand in the center of the abacus and then skip two rods to the left and set their multiplier. This is a slower and more difficult method for blind individuals, and I feel my method is far superior to the latter one.

Examples are the best way to demonstrate how multiplication is performed on the abacus, and I will use this method throughout this and the next section. It is very important that the operator keep his forefinger, right hand in the proper position at all times, so I have gone into great detail to emphasize exactly where the finger should be placed.

Though a person would ordinarily not use the abacus for multiplying one-digit numbers time one-digit numbers, I will give a few examples in

order to show the proper placing of problems and their products.

Example 1---4 x 6 = 24.

Step 1: Set 4 on the first rod to the left, skip two rods, and set 6 to the right, resting your finger on the rod immediately to the right of the 6.

Step 2: Mentally multiply 6 x 4 setting the product 24 on the two rods immediately to the right of the 6. Clear the multiplier 6. If you have followed this procedure correctly, you should now have 4 set on the first rod to the left and the product 24 set on the fifth and sixth rods. Clear the abacus.

Example 2---2 x 4 = 8.

Step 1: Set 2 on the first rod to the left, skip two rods, and set 4 to the right, resting your finger on the rod immediately to the right of the 4.

Step 2: Mentally multiply 4 x 2 setting the product 8 on the second rod to the right of the 4. Clear the multiplier 4. Clear the abacus.

NOTE: The rule to follow in setting products on the proper rods is simply to set a product of one-digit on the second rod to the right of the multiplier and a two-digit product is set on the first and second rods immediately to the right of the multiplier.

Example 3---6 x 8 = 48.

Step 1: Set 6 to the left, skip two rods and set 8, resting your finger on the rod to the right of the 8.

Step 2: Multiply 8 x 6 setting the product 48 on the two rods immediately to the right of the multiplier 8. Clear the multiplier 8. The abacus now shows a 6 set on the first rod to the left and the product 48 set on rods five and six. Clear the abacus.

NOTE: It is impossible to overstress to the student the importance of resting his finger immediately to the right of the multiplier.

Example 4---3 x 2 = 6.

Step 1: Set 3 to the left, skip two rods, and set 2 to the right, resting your finger on the rod immediately to the right of the 2.

Step 2: Multiply 2 x 3 setting the product 6 on the second rod to the right of the 2. Clear the multiplier 2. Clear the abacus.

Example 5---4 x 36 = 144.

Step 1: Set 4 to the left, skip two rods and set 36, resting your finger on the rod immediately to the right of the 6.

Step 2: Multiply 6 x 4 setting the product 24 on the two rods immediately to the right of the 6. Clear the multiplier 6, resting your finger on that rod.

Step 3: Multiply 3 x 4 setting the product 12 on the two rods immediately to the right of the 3, i.e., the 1 goes on the rod where your finger is resting, and the 2 goes on the rod to the right where there is already a 2 set. Clear the multiplier 3. This gives you a final product of 144 which is the correct answer.

NOTE: When you have finished multiplying by a digit in the multiplier, clear that digit immediately.

Example 6---4 x 12 = 48.

Step 1: Set 4 to the left and 12 to the right, resting your finger on the rod to the right of the 12.

Step 2: Multiply 2 x 4 setting the product 8 on the second rod to the right of the 2. Clear the multiplier 2, resting your finger on that rod.

Step 3: Multiply 1 x 4 setting the product 4 on the second rod to the right of the 1. Clear the multiplier 1, leaving the final product of 48.

Example 7---4 x 27 = 108.

Step 1: Set 4 to the left and 27 to the right, resting your finger to the right of the 7.

Step 2: Multiply 7 x 4 setting the product 28 to the right of the 7. Clear the 7 and rest your finger.

Step 3: Multiply 2 x 4 setting the product 8 on the second rod to the right of the 2. Clear the 2, leaving the final product of 108.

Example 8---4 x 72 = 288.

Step 1: Set the problem on the abacus, resting your finger to the right of the 2.

Step 2: Multiply 2 x 4 setting the product 8 on the second rod to the right of the 2. Clear the 2, resting your finger there.

Step 3: Multiply 7 x 4 setting the product 28 on the two rods to the right of the 7. Clear the 7, leaving the final product of 288.

Example 9---6 x 234 = 1,404.

Step 1: Set the problem with the 6 to the left.

Step 2: Multiply 4 x 6 setting the product 24 to the right of the 4. Clear the 4, resting your finger there.

Step 3: Multiply 3 x 6 setting the product 18 on the two rods immediately to the right of the 3. Clear the 3, resting your finger on that rod.

Step 4: Multiply 2 x 6 setting the product 12 to the right of the 2. Clear the 2, leaving the final product of 1,404.

NOTE: I provided many practice problems of this type and suggest that they be used until the student is completely comfortable and confident in his ability to multiply this type of problem.

Exercise I:

1. 4 x 18 = 72
2. 5 x 25 = 125
3. 7 x 13 = 91
4. 6 x 36 = 216
5. 8 x 79 = 632
6. 3 x 88 = 264
7. 5 x 22 = 110
8. 8 x 45 = 360
9. 9 x 43 = 387
10. 6 x 98 = 588

Exercise II:

1. 3 x 724 = 2,172
2. 7 x 463 = 3,241
3. 7 x 146 = 1,022
4. 2 x 327 = 654
5. 5 x 896 = 4,480
6. 6 x 79 = 474
7. 6 x 123 = 738
8. 8 x 109 = 872
9. 9 x 2,468 = 22,212
10. 7 x 5,211 = 36,477

<u>Example 10</u>---23 x 7 = 161.

Step 1: Set 23 to the left, skip two rods and set 7, resting your finger to the right.

Step 2: Multiply 7 x 2 setting the product 14 immediately to the right of the 7, resting the finger below the 4. (NOTE: The multiplier 7 must be multiplied by both the numbers in the multiplicand, and we multiply from the left to the right through the multiplicand.)

Step 3: Multiply 7 x 3 setting the product 21 on the rod where your finger is resting and the one to the right. Clear the multiplier, leaving the product 161 set on the abacus. Clear the abacus.

<u>Example 11</u>---234 x 8 = 1,872.

Step 1: Set the problem with 234 to the left, resting your finger to the right of the 8.

Step 2: Multiply 8 x 2 setting the product 16 to the right of the 8, resting your finger under the 6 in the product.

Step 3: Multiply 8 x 3 setting the product 24 on the rod where your

finger is resting and the rod to the right, resting the finger under the 4 of the product which now reads 184.

Step 4: Multiply 8 x 4 setting the product 32 on the rod where your finger is resting and the rod immediately to the right. Now we have completed multiplying by the 8, so clear the 8. The abacus should now show the final product of 1,872.

NOTE: You do not clear the multiplier until it has been multiplied times all the numbers in the multiplicand from left to right. Also, it is at this point that the student will become much faster and more accurate in operating the abacus if he will read the numbers with his left hand in the multiplicand allowing him to keep his position with the right hand. It is not difficult to read numbers in this manner, and students find that they are able to use the first two fingers on their left hand or their thumb and forefinger. The main thing to stress is that the right hand should keep the proper position at all times in order to prevent setting products on the wrong rods.

In order to eliminate confusion as to where a product will end, especially in the event it ends in a series of zeros, I will introduce a simple rule to follow in making this determination. In the problem 234 x 8, it would have been easy to determine just where the product 1,872 was going to end. Simply rest the finger on the rod to the right of the multiplier, in this case 8, and then move the finger one rod to the right for each digit found in the multiplicand, in this case 3. Move up to the bar locating the unit marker which is immediately to the right of the rod on which the product will end, and in the event there is not a number set on this rod in the final product, then this rod represents a zero. <u>Never</u> think that you will automatically remember where zeros are set,

because you will find yourself making errors in reading your final answers.

Example 12---25 x 4 = 100.

Step 1: After setting the problem, rest the finger on the rod beside the 4. Move the finger two rods to the right, i.e., one for each digit in the multiplicand, and check the bar to find a marker. You find a unit marker immediately to the left of the rod on which your final product will end.

Step 2: After repositioning your finger beside the 4, multiply 4 x 2, setting the product 8 on the second rod, resting your finger on that rod.

Step 3: Multiply 4 x 5 setting the product 20 on the rod on which your finger rests and the rod to the right. Clear the 4, leaving the final product of 100.

NOTE: By determining the rod on which the final product will end, it is easy to see that that rod and the one to the left show no numbers, representing zeros.

Example 13---125 x 8 = 1,000.

Step 1: After setting the problem on the abacus, rest your finger beside the 8. Since there are three digits in the multiplicand, move the finger three rods to the right and check the bar to locate the nearest unit marker.

Step 2: After repositioning your finger beside the 8, multiply 8 x 1 and set the product 8 on the second rod to the right of the multiplier 8, resting your finger on that rod.

Step 3: Multiply 8 x 2 and set the product 16 on the rod where your finger rests and the rod to the right. Rest your finger under the 6.

Step 4: Multiply 8 x 5 and set the 4 of the product 40 on the rod where your finger is resting. Clear the 8, leaving the final product of 1,000.

NOTE: Zeros will take care of themselves if they receive the proper respect as all other numbers, and simply move your finger one rod to the right in the product to represent setting a zero.

Example 14---308 x 7 = 2,156.

Step 1: Set the problem and rest your finger beside the 7. Now check the bar above the third rod to the right of your finger for a unit marker.

Step 2: After repositioning the finger beside the 7, multiply 7 x 3 and set the product 21 immediately to the right of the 7, resting your finger under the 1.

Step 3: Multiply 7 x 0 setting the product 0, which is a one-digit product on the second rod, i.e., the rod to the right of your finger. Rest your finger on that rod.

Step 4: Multiply 7 x 8 and set the product 56 on the rod where your finger is resting and the rod to the right. Clear your multiplier, leaving your final product of 2,156.

Example 15---705 x 6 = 4,230.

Step 1: Set the problem, resting the finger in the proper position. Check the bar above the third rod to the right of your finger to locate the unit marker. The marker is immediately to the right of the rod on which the product will end.

Step 2: Reposition the finger beside the 6 and multiply 6 x 7 setting the product 42 immediately to the right of the 6. Rest the finger under the 2.

Step 3: Multiply 6 x 0 setting the product 0 on the rod to the right, resting your finger on that rod.

Step 4: Multiply 6 x 5 setting the product 30 on the rod on which your finger is resting and the rod to the right. Clear your multiplier, leaving your final product of 4,230.

Example 16---36 x 74 = 2,664.

Step 1: Set 36 to the left, skip two rods, set 74 to the right, resting your finger to the right of the 4. Check the bar two rods to the right to locate your unit marker, and reposition the finger beside the 4.

Step 2: Multiply 4 x 3 setting the product 12 immediately to the right of the 4, resting your finger below the 2.

Step 3: Multiply 4 x 6 setting the product 24 on the rod on which your finger rests and the one to the right. Since we have finished multiplying with the 4, clear it, resting your finger on that rod.

Step 4: Multiply 7 x 3 setting the product 21 immediately to the right of the 7. The finger should come to rest under the second 2 showing in the product.

Step 5: Multiply 7 x 6 setting the product 42 on the rod where your finger rests and the rod to the right. Clear the 7, and this leaves the final product of 2,664.

Example 17---234 x 67 = 15,678.

Step 1: Set the problem with the 234 to the left. With your finger resting on the rod to the right of the 7, check the unit marker three rods to the right to see where your product will end.

Step 2: With the finger again on the rod beside the 7, multiply 7 x 2 and set the product 14 immediately to the right. Rest your finger below the 4.

Step 3: Multiply 7 x 3 and set the product 21 on the rod on which your finger rests and the one to the right, resting your finger under the 1.

Step 4: Multiply 7 x 4 and set the product 28 on the rod where your finger rests and the rod to the right. Clear the 7 and rest your finger on that rod.

Step 5: Multiply 6 x 2 and set the product 12 immediately to the right of the 6. Rest your finger under the 3.

Step 6: Multiply 6 x 3 and set the product 18 on the rod on which your finger rests and the rod to the right. Let your finger rest under the 4 of the product.

Step 7: Multiply 6 x 4 and set the product 24 on the rod on which your finger rests and the rod to the right. Clear the 6, leaving the final product of 15,678.

<u>Example 18</u>---206 x 3005 = 619,030.

Step 1: Set the problem with 206 to the left. Place your finger on the rod immediately to the right of the 5 in the multiplier, then check the third rod to the right to locate the unit marker. You can easily see in this case that your product will terminate on the last rod to the right.

Step 2: Reposition your finger and multiply 5 x 2, setting the product 10 on the two rods to the right of the 5. Rest your finger on the rod where the zero is set.

Step 3: Multiply 5 x 0 and set the product 0 on the rod to the right of your finger and rest your finger on that rod.

Step 4: Multiply 5 x 6 and set the 3 of the product 30 on the rod where your finger rests. Now clear the 5 from the multiplier and then simply move your hand to the left until your finger rests on the rod

immediately to the right of the 3 in the multiplier, and rest your finger on that rod.

Step 5: Multiply 3 x 2 and set the product 6 on the second rod to the right of the 3, resting your finger on that rod.

Step 6: Multiply 3 x 0 and set the product 0 on the rod to the right of your finger. Rest your finger on that rod.

Step 7: Multiply 3 x 6 and set the product 18 on the rod where your finger is resting and the rod to the right. Clear the 3, leaving the final product of 619,030.

Example 19---576 x 1,234 = 710,784.

Step 1: Set the problem with the 576 to the left. On checking the rod to determine where the product will end, you find that it will end on the last rod to the right. Rest your finger beside the 4 and begin multiplying.

Step 2: Multiply 4 x 5 and set the product 20 immediately to the right of the 4, letting the finger rest on the rod with the 0.

Step 3: Multiply 4 x 7 and set the product 28 on the rod where your finger is resting and the rod to the right. Rest your finger under the 8.

Step 4: Multiply 4 x 6 and set the product 24 on the rod where your finger is resting and the rod to the right. Clear the 4 from the multiplier, resting the finger on that rod.

Step 5: Multiply 3 x 5 and set the product 15 immediately to the right of the 3, resting your finger under the 7.

Step 6: Multiply 3 x 7 and set the product 21 on the rod where your finger is resting and on the rod to the right, resting your finger under the 4 on that rod.

Step 7: Multiply 3 x 6 and set the product 18 on the rod where your finger is resting and the rod to the right. Clear the 3 from the multiplier, resting your finger on that rod.

Step 8: Multiply 2 x 5 and set the product 10 on the two rods to the right of the 2, resting your finger on the rod with the second 1.

Step 9: Multiply 2 x 7 and set the product 14 on the rod where your finger is resting and the rod to the right, resting your finger on this rod below the 3.

Step 10: Multiply 2 x 6 and set the product 12 on the rod where your finger is resting and the rod to the right. Clear the 2 from the multiplier, resting your finger on that rod.

Step 11: Multiply 1 x 5 and set the product 5 on the second rod to the right. Rest your finger below the 6 formed on this rod.

Step 12: Multiply 1 x 7 and set the product 7 on the rod to the right of your finger, resting your finger on that rod.

Step 13: Multiply 1 x 6 and set the product 6 on the rod to the right of your finger. Clear the 1 from the multiplier, leaving the final product of 710,784.

By following the simple rules listed below, you can complete any multiplication problem:

1. Set the number on the abacus from left to right as it is being read, multiplying by the number set to the right.

2. Determine where the product will terminate, then position the finger on the first rod immediately to the right of the multiplier.

3. Multiply each number in the multiplicand, starting on the left end of it, by each number in the multiplier, starting on the right end of

it. The finger is always used to keep the proper position on the rods. Be sure to always give zeros in the multiplicand the same respect you give the other numbers, and move the finger one rod to the right each time a zero appears in the multiplicand. Zeros in the multiplier will always take care of themselves.

4. It is a good practice to always set the smaller numbers, i.e., the ones with the smaller amount of digits, to the left on the abacus. This will permit you to work longer problems. Do not do this in place of the first rule of setting numbers as they are called out. If you are arranging your own numbers or reading them from a book, you might find it to your advantage to set the smaller numbers to the left.

A good practice exercise is to allow students to multiply 123,456,789 by 9 or a multiple of it such as 18, 27, 36, etc. I have given the answer to all these different combinations in the exercise in the back of the book. You can easily see that it will be necessary for students, in working these problems, to "remember" the multiplicand and not set it on the abacus for lack of space. It is a good policy to try to learn to remember the multiplicand when it is a simple one, but beginning students should never be encouraged to do this.

The following practice problems should provide ample exercise in learning to multiply with ease and accuracy. The teacher should continue to give encouragement and support until the student has gained confidence in his own ability to operate the abacus without error.

Exercise I:

1. 12 x 34 = 408
2. 23 x 45 = 1,035
3. 34 x 56 = 1,904

Exercise II:

1. 48 x 876 = 42,048
2. 21 x 26 = 546
3. 43 x 18 = 774

4. 18 x 72 = 1,296
5. 47 x 68 = 3,196
6. 63 x 236 = 14,868
7. 17 x 543 = 9,231
8. 61 x 161 = 9,821
9. 32 x 671 = 21,472
10. 18 x 789 = 14,202

4. 87 x 112 = 9,744
5. 63 x 142 = 8,946
6. 22 x 362 = 7,964
7. 24 x 234 = 5,616
8. 24 x 11,101 = 266,424
9. 29 x 5,632 = 163,328
10. 91 x 6,502 = 591,682

Exercise III:

1. 302 x 618 = 186,636
2. 200 x 675 = 135,000
3. 415 x 627 = 260,205
4. 928 x 323 = 299,744
5. 214 x 704 = 150,656
6. 312 x 407 = 126,984
7. 518 x 209 = 108,262
8. 818 x 402 = 328,836
9. 129 x 870 = 112,230
10. 119 x 801 = 95,319

DIVISION

Before going directly into division, it is quite often helpful to the student to review subtraction at this point. This does not need to be done on a routine basis, but if the student appears to need the review, it would be a good place for it.

Division on the abacus is not difficult since it follows the same procedure used in braille or print.

The abacus, being a sophisticated, mechanical device, eliminates the need to perform strenuous mental calculations in division. Do not allow yourself to "figure out" answers in your head, but allow the abacus to give you the proper answers. As in the previous processes of calculation, the forefinger of the right hand plays an important role in division. It is important that this finger be positioned on the proper rod at all times and should not be removed from the abacus until the problem has been completed. Examples will be used to demonstrate proper techniques, and emphasis will be placed on positioning the finger in the proper place.

Division problems are set on the abacus from left to right, beginning with the first rod to the left. Set the divisor to the left and the dividend to the right, leaving a total of four unused rods between them. This is similar to the way division problems are set up in print or braille, but since it is impossible to set our quotient above the dividend on the abacus, as it is done with the other methods, it is necessary to set the quotient on one of the two rods immediately to the left of the dividend. It will be made clear through the following examples exactly which rod should be used for the quotient.

<u>Example 1</u>---6 ÷ 2 = 3.

Step 1: Set the divisor 2 on the first rod to the left, skip four rods, set the dividend 6.

Step 2: Mentally divide 2 into 6 setting the quotient 3 on the second rod to the left of the 6 in the dividend. Rest the finger on the rod immediately to the right of the quotient 3.

Step 3: Multiply 3 x 2 and subtract the product 6 from the 6 in the dividend.

NOTE: To determine whether your quotient is set on the first rod or second rod to the left of the dividend, follow this simple rule--skip a rod if your divisor will divide into one digit of your dividend, but do not skip a rod if your divisor must be divided into two or more digits of the dividend. In the previous example, the divisor 2 was divided into the one-digit 6, so it was necessary to skip a rod in setting the quotient. In the next example, 12 ÷ 6, however, it will not be necessary to skip a rod in setting the quotient 2.

<u>Example 2</u>---12 ÷ 6 = 2.

Step 1: Set 6 to the left, skip four rods, and set 12.

Step 2: Mentally divide 6 into 12 and set the quotient 2 on the rod immediately to the left of the dividend. Rest the finger on the rod immediately to the right of the quotient 2, i.e., on the rod below the 1 of the dividend.

Step 3: Multiply 2 x 6 subtracting the product 12 from the 12 of the dividend.

NOTE: Since it was necessary to divide the divisor 6 into two digits of the dividend, the quotient had to be set immediately to the left of the dividend without skipping a rod.

Another way of determining which rod is to be used in setting your quotient is the "four-S" rule, same-size-smaller-skip. If the number in the divisor is the same size or smaller than the number in the dividend, then skip a rod. If it is not, you do not need to skip. In the first example, the divisor 2 was smaller than the 6 into which it was divided, so it was necessary to skip a rod. In the second example, however, 6 is not the same size or smaller than the 1 of the dividend, so it was not necessary to skip a rod. Rather than adopting just one of these methods of determining the proper rod, both methods should be used by the teacher. The student should decide which method makes more sense to him. Throughout this text, I will use the first method listed above.

<u>Example 3</u>---18 ÷ 9 = 2.

Step 1: Set 9 to the left and 18 to the right.

Step 2: Divide 9 into 18 setting the quotient 2 immediately to the left of the 18 and rest the finger on the rod to the right, i.e., the rod on which the 1 of your dividend is set.

Step 3: Multiply the quotient 2 x 9 subtracting the product 18 from the 18 in the dividend.

<u>Example 4</u>---497 ÷ 7 = 71.

Step 1: Set the problem on the abacus with the 7 to the left.

Step 2: Divided 7 into 49, setting the quotient on the rod immediately to the left of the dividend, and rest your finger to the right on the rod below the 4.

Step 3: Multiply 7 x 7 subtracting the product 49 from the 49 in the dividend.

Step 4: Divide the 7 into the 7 setting the quotient 1 on the second rod to the left of the dividend.

Step 5: Multiply the quotient 1 x 7, subtracting the product 7 from the 7 in the dividend.

Example 5---2,868 ÷ 6 = 478.

Step 1: Set the problem on the abacus with the 6 to the left.

Step 2: Divide 6 into 28 setting the quotient 4 on the rod immediately to the left of the dividend. Rest the finger on the rod to the right below the 2 of the dividend.

Step 3: Multiply the quotient 4 x 6 subtracting the product 24 from the 28 in the dividend.

Step 4: Divide 6 into 46 setting the quotient 7 immediately to the left. Again, rest your finger on the rod below the 4.

Step 5: Multiply the quotient 7 x 6, subtracting the product 42 from the 46 in the dividend.

Step 6: Divide 6 into 48, setting the quotient 8 immediately to the left. Rest your finger below the 4 of the dividend.

Step 7: Multiply the quotient 8 x 6, subtracting the product 48 from the 48 in the dividend. This leaves the final quotient of 478 on the abacus.

Example 6---80,072 ÷ 8 = 10,009.

Step 1: Set the problem on the abacus with the 8 to the left.

Step 2: Divide 8 into 8, setting the quotient 1 on the second rod to the left of the dividend. Rest your finger on the rod to the right of the 1.

Step 3: Multiply 1 x 8 subtracting the product 8 from the 8 in the dividend.

Step 4: Since 8 will not divide into the two zeros of the dividend, divide it into 72, setting the quotient 9 immediately to the left of the 7. Rest your finger under the 7.

Step 5: Multiply 9 x 8, subtracting the product 72 from the 72 in the dividend and leaving the final quotient of 10,009.

NOTE: As you can see, zeros will take care of themselves if you set the quotient on its proper rod.

<u>Example 7</u>---8,928 ÷ 6 = 1,488.

Step 1: Set the problem.

Step 2: Divide 6 into 8 setting the product 1 on the second rod to the left of the dividend. Rest the finger on the empty rod to the right.

Step 3: Multiply 1 x 6, subtracting the product 6 from the 8 in the dividend.

Step 4: Divide 6 into 29, setting the quotient 4 immediately to the left of the dividend. Rest your finger on the rod to the right, i.e., below the 2.

Step 5: Multiply 4 x 6, subtracting the product 24 from the 29 in the dividend.

Step 6: Divide 6 into 52, setting the quotient 8 immediately to the left of the dividend. Rest the finger on the rod to the right.

Step 7: Multiply 8 x 6 subtracting the product 48 from the 52 of the dividend.

Step 8: Divide 6 into 48 setting the quotient 8 immediately to the left, and rest your finger on the rod to the right.

Step 9: Multiply 8 x 6, subtracting the product 48 from the 48 in the dividend, leaving the final product of 1,488.

NOTE: Remember that each time a new digit is added to the quotient, you should rest your finger on the rod to the right. This is because your quotient becomes a multiplier and you always set your finger to the right of the multiplier.

Example 8---456 ÷ 3 = 152.

Step 1: Set the problem.

Step 2: Divide the 3 into 4 setting the quotient 1 on the second rod to the left. Rest your finger on the rod to the right.

Step 3: Multiply 1 x 3 subtracting the product 3 from the 4 in the dividend.

Step 4: Divide 3 into 15, setting the quotient 5 immediately to the left, resting your finger to the right.

Step 5: Multiply 5 x 3, subtracting the product 15 from the 15 in the dividend.

Step 6: Divide 3 into 6, setting the quotient 2 on the second rod to the left. Rest your finger to the right.

Step 7: Multiply 2 x 3, subtracting the product 6 from the dividend, leaving your final quotient 152.

NOTE: The following example demonstrates a division problem in which there is a remainder in the quotient. This is very common in division, and you will see that it is easy to recognize the remainder.

Example 9---637 ÷ 4 = 159 r 1.

Step 1: Set the problem.

Step 2: Divide 4 into 6, setting the quotient 1 on the second rod to the left. Rest your finger to the right.

Step 3: Multiply 1 x 4, subtracting the product 4 from the 6 in the dividend.

Step 4: Divide 4 into 23, setting the quotient 5 immediately to the left of the dividend, and rest your finger to the right.

Step 5: Multiply 5 x 4, subtracting the product 20 from the 23 in the dividend.

Step 6: Divide 4 into 37, setting the quotient 9 immediately to the left of the dividend. Rest your finger to the right.

Step 7: Multiply 9 x 4, subtracting the product 36 from the 37 in the dividend, leaving the remainder of 1. Since you can not divide 4 into 1, you have your final quotient of 159 with the remainder of 1.

NOTE: In the event the dividend ends in a zero, be sure to check your unit marker to determine on which rod the zero is placed. Be sure to consider this zero in working the problem.

Example 10---72,040 ÷ 8 = 9,005.

Step 1: Set the problem.

Step 2: Divide 8 into 72, setting the quotient 9 immediately to the left. Rest your finger to the right.

Step 3: Multiply 9 x 8, subtracting the product 72 from the 72 in the dividend.

Step 4: Divide 8 into 40, setting the quotient 5 immediately to the left of the dividend. Rest your finger to the right of the 5.

Step 5: Multiply 5 x 8, subtracting the product 40 from the 40 in the dividend. This leaves your final quotient of 9,005.

NOTE: Had you not remembered that this problem ended with a zero, you would have come out with the incorrect quotient of 900 with the remainder of 4.

You should now be able to complete the following division exercises. If you have difficulty, review the rules on positioning the quotient and then review some of the examples.

Exercise I:

1. 872 ÷ 8 = 109
2. 474 ÷ 6 = 79

Exercise II:

1. 9,736 ÷ 8 = 1,217
2. 111,105 ÷ 9 = 12,345

3. 25,873 ÷ 4 = 6,468 r1
4. 51,075 ÷ 5 = 10,215
5. 321,006 ÷ 3 = 107,002
6. 63,018 ÷ 9 = 7,002
7. 12,345 ÷ 9 = 1,371 r6
8. 60,124 ÷ 2 = 30,062
9. 36,477 ÷ 7 = 5,211
10. 38,297 ÷ 5 = 7,659 r2

3. 87,654 ÷ 4 = 21,913 r2
4. 1,985 ÷ 5 = 397
5. 1,984,234 ÷ 7 = 283,462
6. 169,696 ÷ 8 = 21,212
7. 738 ÷ 6 = 123
8. 252,960 ÷ 4 = 63,240
9. 1,000 ÷ 5 = 200
10. 56 ÷ 4 = 14

LONG DIVISION

(Division by Two or More Digit Numbers)

In short division, we multiplied each new digit of the quotient times the divisor. In long division, the process is the same, except that it is necessary to multiply the digits in the quotient times all the digits in the divisor, working from left to right. Being able to keep your proper position with the right hand becomes of utmost importance in long division. Therefore, I will emphasize the exact positioning throughout the following examples.

Example 1---128 ÷ 64 = 2.

Step 1: Set the problem with the divisor 64 to the left.

Step 2: Divide the 6 of the divisor into the 12 of the dividend, and set the quotient 2 immediately to the left of the dividend, resting your finger to the right, i.e., on the rod where the 1 of the dividend is set.

Step 3: Multiply the quotient 2 x 6, subtracting the product 12 from the 12 of the dividend and rest your finger on the rod from which the 2 was subtracted.

Step 4: Multiply the quotient 2 x 4, subtracting the product 8 from the 8 of the dividend. This leaves the final quotient of 2.

Example 2---529 ÷ 23 = 23.

Step 1: Set the problem with the divisor 23 to the left.

Step 2: Divide 2 into 5, setting the quotient 2 on the second rod to the left. Rest your finger to the right of the quotient.

Step 3: Multiply the quotient 2 x 2, subtracting the product 4 from the 5 in the dividend, resting your finger on that rod.

Step 4: Now multiply the quotient 2 x 3 in the divisor subtracting the product 6 from the 12 in the dividend.

Step 5: Divide 2 into 6, and set the quotient 3 on the second rod to the left. Rest your finger on the rod to the right.

Step 6: Multiply the quotient number 3 x 2 in the divisor, subtracting the product 6 from the 6 in the dividend, resting your finger on the rod.

Step 7: Multiply the quotient 3 x 3 subtracting the product 9 from the 9 in the dividend. This leaves your final quotient of 23.

NOTE: In the example 128 ÷ 64 = 2, the 6 of the divisor would not go into the first digit 1 of the dividend, but had to be divided into two digits; thus, it was necessary to set the quotient immediately to the left of the dividend.

In the example 529 ÷ 23 = 23, (steps 2 and 5) the divisor 2 would divide into the first digit of the dividend.

<u>Example 3</u>---2,432 ÷ 76 = 32.

Step 1: Set the problem on the abacus.

Step 2: Divide 7 into 24, setting the quotient 3 immediately to the left. Rest your finger on the rod to the right.

Step 3: Multiply 3 x 7 subtracting the product 21 from the 24 of the dividend, resting your finger on the rod from which the 1 was subtracted.

Step 4: Multiply 3 x 6, subtracting the product 18 from the 33 in the dividend.

Step 5: Divide 7 into 15, setting the quotient 2 immediately to the left. Rest your finger under the 1 of the dividend.

Step 6: Multiply 2 x 7, subtracting the product 14 from the 15 in the dividend, resting your finger on the second rod, i.e., the one from which

the 4 was subtracted.

Step 7: Multiply 2 x 6, subtracting the product 12 from the 12 of the dividend, leaving your final quotient of 32.

Example 4---6,762 ÷ 42 = 161.

Step 1: Set the problem on the abacus.

Step 2: Divide 4 into 6, setting the quotient 1 on the second rod to the left. Rest your finger on the rod to the right.

Step 3: Multiply 1 x 4, subtracting the product 4 from the 6 in the dividend, resting your finger on that rod.

Step 4: Multiply 1 x 2, subtracting that product of 2 from the 7 in the dividend.

Step 5: Divide 4 into 25, setting the quotient 6 on the rod immediately to the left. Rest your finger to the right.

Step 6: Multiply 6 x 4, subtracting the product 24 from the 25 of the dividend. Rest your finger on the rod below the 1 of the dividend.

Step 7: Multiply 6 x 2, subtracting the product 12 from the 16 of the dividend, leaving a new dividend of 42.

Step 8: Divide 4 into 4, and since it is the same, check the second digit of both the divisor and dividend. Since they are the same, set the quotient 1 on the second rod to the left. Rest your finger to the right of the quotient.

Step 9: Multiply 1 x 4, subtracting the product 4 from the 4 in the dividend, resting your finger on that rod.

Step 10: Multiply 1 x 2, subtracting the product 2 from the 2 remaining in the dividend, leaving the final quotient of 161.

NOTE: In the event the first digit of the divisor is the same as the first digit of the dividend, then it is necessary to compare the second two

digits, and if they are the same or the divisor is smaller, then set the quotient 1 on the second rod to the left. The following example will illustrate this method.

Example 5---144 ÷ 12 = 12.

Step 1: Set the problem.

Step 2: Divide 1 into 1, and since they are the same, compare the 2 of the divisor to the 4 of the dividend. Since it is smaller, set your quotient 1 on the second rod to the left. Rest your finger to the right of the quotient.

Step 3: Multiply 1 x 1, subtracting the product 1 from the 1 in the dividend. Rest your finger on that rod.

Step 4: Multiply 1 x 2, subtracting the product 2 from the 4 of the dividend.

Step 5: Divide 1 into 2 setting the quotient 2 on the second rod to the left, resting your finger to the right.

Step 6: Multiply 2 x 1 subtracting the product 2 from the 2 in the dividend, resting your finger on that rod.

Step 7: Multiply 2 x 2 and subtract the product 4 from the remaining 4 in the dividend, leaving the final quotient of 12.

NOTE: The following problems can be used for additional practice.

Exercise I:

1. 64 ÷ 32 = 2
2. 546 ÷ 26 = 21
3. 774 ÷ 43 = 18
4. 9,744 ÷ 87 = 112
5. 8,946 ÷ 63 = 142
6. 90,675 ÷ 45 = 2,015
7. 7,964 ÷ 362 = 22
8. 5,616 ÷ 234 = 24
9. 993,531 ÷ 847 = 1,173
10. 98,406 ÷ 426 = 231

USE OF NINES IN DIVISION

In example 5, it was demonstrated that in the event the first digit of the divisor and dividend are the same, it is necessary to compare the second digits. If the second digit in the divisor is the same size or smaller, you set the quotient 1 on the second rod to the left and multiply. In the event, however, that the second digit of the divisor is larger than the second digit of the dividend, it is a good idea to set the quotient 9 and multiply it times the divisor. The quotient then would be set immediately to the left of the dividend. The following two examples will demonstrate this principle, though the experienced operator soon learns to determine whether the 9 should be used or not.

<u>Example 6</u>---2,184 ÷ 24 = 91.

Step 1: Set the problem.

Step 2: Divide 2 into 2, and since they are the same, compare the second digits of both divisor and dividend. Since the 4 in the divisor is larger than the 1 in the dividend, set the quotient 9 immediately to the left, and rest your finger on the rod to the right.

Step 3: Multiply 9 x 2, subtracting the product 18 from the 21 in the dividend. Rest your finger below the 3.

Step 4: Multiply 9 x 4, subtracting the product 36 from the 38 of the dividend.

Step 5: Divide 2 into 2, and since they are the same, compare the second digits, 4. Since they are the same, set the quotient 1 on the second rod to the left, resting your finger to the right of it.

Step 6: Multiply 1 x 2, subtracting the product 2 from the 2 in the

dividend. Rest your finger on that rod.

Step 7: Multiply 1 x 4, subtracting the product 4 from the remaining 4 of the dividend, leaving the final quotient of 91.

Example 7---30,668 ÷ 34 = 902.

Step 1: Set the problem.

Step 2: Since you immediately see that the first digits of the divisor and dividend are the same, compare the second digits. The divisor 4 is larger than the 0 in the dividend, so set the quotient 9 immediately to the left of the dividend. Rest your finger to the right.

Step 3: Multiply 9 x 3, subtracting the product 27 from the 30 of the dividend, resting the finger below the 3.

Step 4: Multiply 9 x 4, subtracting the product 36 from the 36 of the dividend.

Step 5: Divide 3 into 6, setting the quotient 2 on the second rod to the left, resting your finger to the right.

Step 6: Multiply 2 x 3 subtracting the product 6 from the 6 of the dividend, resting your finger on that rod.

Step 7: Multiply 2 x 4, subtracting the product 8 from the remaining 8 in the dividend, leaving the final quotient of 902.

NOTE: You sometimes find that when you have tried a quotient number, i.e., 9 in the previous two examples, that you have used a quotient too large, and it will be necessary to make a revision. This simply means that you try a smaller quotient number, but it is not necessary to reset the original dividend and start over. Instead, simply follow the procedures illustrated in the following examples, and the abacus will correct itself.

DOWNWARD REVISION

Example 8---152 ÷ 19 = 8.

Step 1: Set the problem.

Step 2: Since the first digit of the divisor and the dividend are the same, compare the second digits. Since the 9 is larger than the 5, set the quotient 9 to the left. Rest your finger to the right.

Step 3: Multiply 9 x 1, subtracting the product 9 from the 15 in the dividend, resting your finger below the 6.

Step 4: Multiply 9 x 9 and you quickly see that it is impossible to subtract the product 81 from the 62 in the dividend, so it will be necessary to revise the quotient 9.

Step 5: Subtract 1 from the quotient 9, leaving the new quotient 8 set on that rod, resting your finger on the rod to the right of the 8.

Step 6: Multiply the one in the divisor by the 1, i.e., the difference between the quotient 9 and the quotient 8, and add this product 1 to the rod to the right of your finger. This is because the product 1 is a one-digit number. Rest your finger on that rod.

Step 7: Multiply the new quotient 8 x 9 of the divisor, subtracting the product 72 from the 72 in the dividend.

NOTE: In revision, multiply the difference between the two quotient numbers which you try, times the numbers in the divisor which have been used, and add this product back into the dividend on the proper rods.

Example 9---37,278 ÷ 57 = 654.

Step 1: Set the problem.

Step 2: Divide 5 into 37, and it appears to go 7 times, so set the

quotient 7 immediately to the left, and rest your finger to the right.

Step 3: Multiply 7 x 5, subtracting the product 35 from the 37 in the dividend. Rest your finger below the 2 of the dividend.

Step 4: Multiply 7 x 7, and you quickly realize that you can not subtract the product 49 from the 22 of the dividend.

Step 5: Since 7 is too large a quotient, subtract 1 from it, leaving the new quotient 6, and rest your finger to the right.

Step 6: Multiply 1, i.e., the difference between the quotient 7 and the 6, times 5 in the divisor, adding the product 5 to the rod to the right of your finger. Rest your finger on that rod.

Step 7: Multiply 6 x 7, subtracting the product 42 from the 72 in the dividend.

Step 8: Divide 5 into 30, and set the quotient 6 immediately to the left, resting your finger under the 3.

Step 9: Multiply 6 x 5, subtracting the product 30 from the 30 in the dividend. Rest your finger on that rod.

Step 10: Multiply 6 x 7, and it is impossible to subtract the product 42 from the 07 of the dividend.

Step 11: Revise the quotient 6 to 5, resting your finger to the right.

Step 12: Multiply 1, i.e., the difference between the original quotient 6 and the new quotient 5, times the 5 in the divisor, adding the product 5 to the dividend, to the right of your finger. Rest your finger on that rod.

Step 13: Multiply 5 x 7, subtracting the product 35 from the 57 of the dividend.

Step 14: Divide 5 into 22, setting the quotient 4 immediately to the left of the dividend. Rest your finger on the rod to the right.

Step 15: Multiply 4 x 5, subtracting the product 20 from the 22 in the

dividend, resting your finger on the rod below the 2.

Step 16: Multiply 4 x 7, subtracting the product 28 from the 28 in the dividend, leaving the final quotient of 654.

Example 10---1,422 ÷ 18 = 79.

Step 1: Set the problem.

Step 2: Since the first digit of the divisor and dividend are both ones, compare the second digits. The 8 in the divisor is larger than the 4 in the dividend, so set the quotient 9 immediately to the left, and rest your finger to the right.

Step 3: Multiply 9 x 1, subtracting the product 9 from the 14 in the dividend. Now rest your finger below the 5 in the dividend.

Step 4: Multiply 9 x 8, and you see that the product 72 will not subtract from the 52 of the dividend, so it is necessary to revise your quotient.

Step 5: Subtract 1 from the quotient 9 and rest your finger to the right.

Step 6: Multiply 1 x 1, adding the product 1 to the 5 in the dividend, resting your finger on that rod.

Step 7: Multiply the quotient 8 x 8 in the divisor, and you see that it is necessary to revise your quotient again. Subtract 1 from the 8 in the quotient, and rest your finger to the right.

Step 8: Multiply the 1 you subtracted times the 1 in the divisor, adding the product 1 to the 6 in the dividend, resting your finger on that rod.

Step 9: Multiply the new quotient 7 times 8 of the divisor, subtracting the product 56 from the 72 of the dividend, leaving a new dividend of 162.

Step 10: Since the first digit in the divisor and dividend are both ones, it is again necessary to compare the second digits, and you will find that the 8 in the divisor is larger than the 6 in the dividend, so again set the quotient 9 immediately to the left and rest your finger to the right.

Step 11: Multiply 9 x 1, subtracting the product 9 from the 16 in the dividend. Rest your finger below the 7.

Step 12: Multiply 9 x 8, subtracting the product 72 from the 72 in the dividend, leaving the final quotient of 79.

Example 11---924 ÷ 28 = 33.

Step 1: Set the problem.

Step 2: Divide 2 into 9, setting the quotient 4 on the second rod to the left. Rest your finger to the right.

Step 3: Multiply 4 x 2, subtracting the product 8 from the 9 in the dividend, resting your finger on that rod.

Step 4: Multiply 4 x 8, and you see that you can not subtract the product 32 from the 12 of the dividend, so you subtract 1 from the quotient 4, resting your finger to the right.

Step 5: Multiply the 1 you subtracted times the 2 in the divisor, setting the product 2 on the rod to the right of your finger. Rest your finger on that rod.

Step 6: Multiply 3 x 8, subtracting the product 24 from the 32 in the dividend.

Step 7: Divide 2 into 8, setting the quotient 4 on the second rod to the left, resting your finger to the right.

Step 8: Multiply 4 x 2, subtracting the product 8 from the 8 in the dividend. Rest your finger on that rod.

Step 9: Multiply 4 x 8, and it is impossible to subtract the product 32 from the 4 remaining in the dividend, so it is necessary to revise the quotient to 3. Rest your finger to the right.

Step 10: Multiply the 1 you subtracted times the 2 in the divisor, setting the product 2 on the rod to the right of your finger, resting your finger on that rod.

Step 11: Multiply the new quotient 3 x 8, subtracting the product 24 from the 24 in the dividend. This leaves your final quotient of 33.

NOTE: As I stated earlier, you will learn from experience which quotient number to try first. In the event you try a quotient number too small, rather than too large, you will find it necessary to revise your quotient upward rather than downward. The following example will demonstrate this procedure.

Example 12---793 ÷ 13 = 61.

Step 1: Set the problem.

Step 2: It would appear that 1 would divide into 7, seven times. Try 5, however, setting it on the second rod to the left. Rest your finger to the right.

Step 3: Multiply 5 x 1, subtracting the product 5 from the 7 in the dividend. Rest your finger on that rod.

Step 4: Multiply 5 x 3 subtracting the product 15 from the 29 in the dividend. This leaves a remainder on these two rods of 14 which is larger than the divisor. It is necessary, therefore, to revise your quotient.

Step 5: Simply divide the 13 of the divisor into the 14 of the dividend, setting the quotient 1 on the second rod to the left, i.e., where the previous quotient 5 was set. This gives the quotient of 6 on that rod,

but you must multiply the divisor by only the new quotient 1, i.e., the number you added in the revision. Rest your finger to the right of the quotient and multiply.

Step 6: Multiply 1 x 1, subtracting the product 1 from the 1 in the dividend, resting your finger on that rod.

Step 7: Multiply 1 x 3 and subtract the product 3 from the 4 in the dividend.

Step 8: Divide 13 into 13, setting the quotient 1 on the second rod to the left. Rest your finger to the right.

Step 9: Multiply 1 x 1, subtracting the product 1 from the 1 in the dividend, resting your finger on that rod.

Step 10: Multiply 1 x 3, subtracting the product 3 from the 3 in the dividend, leaving the final quotient of 61.

NOTE: In revising upward, simply divide the divisor into the remaining dividend following the rules of setting the quotient on the proper rod, and multiply the divisor by the quotient, which you will always be adding to another number, then subtract the product of that multiplication from the dividend. It takes no mental calculation to revise upward.

LONG DIVISION WITH ZEROS

Zeros do not offer any difficulty in carrying out division on the abacus. It is simply necessary to note when and where zeros are set, and give them the same respect you give any other number. The following two examples will demonstrate the procedure to follow.

Example 13---73,800 ÷ 36 = 2,050.

Step 1: Set the problem, noting where the final zero of the dividend terminates. In this case, there should be a unit marker immediately to the left of the rod on which it ends.

Step 2: Divide 3 into 7, setting the quotient 2 on the second rod to the left, resting the finger to the right.

Step 3: Multiply 2 x 3, subtracting the product 6 from the 7 in the dividend, resting your finger on that rod.

Step 4: Multiply 2 x 6, subtracting the product 12 from the 13 in the dividend.

Step 5: Divide 3 into 18, setting the quotient 5 immediately to the left of the dividend. Rest your finger to the right.

Step 6: Multiply 5 x 3, subtracting the product 15 from the 18 in the dividend. Rest your finger below the 3.

Step 7: Multiply 5 x 6, subtracting the product 30 from the 30 in the dividend.

Step 8: Since your finger is resting on the rod immediately to the left of the unit marker, and since we originally determined that there was a zero set on the rod to the right of this marker, then it is necessary to divide 36 into zero, and remember to set a zero following the 5 in the

quotient, leaving the final quotient of 2,050.

NOTE: In step 5, it would have appeared that 3 would have gone into 18 six times, but I preferred to try the quotient 5 and would have made an upward revision had it been necessary. Upward revisions are preferred over downward revisions since they are self-correcting procedures that take no mental calculation.

Example 14---7,000 ÷ 20 = 350.

Step 1: Set the problem, noting on which rod the final zero will be set.

Step 2: Divide 2 into 7, setting the quotient 3 on the second rod to the left, resting your finger on the rod to the right.

Step 3: Multiply 3 x 2, subtracting the product 6 from the 7 in the dividend, resting your finger on that rod.

Step 4: Multiply 3 x 0, setting the product 0 on the rod to the right of your finger.

Step 5: Divide 2 into 10, setting the quotient 5 immediately to the left of the dividend, resting your finger to the right.

Step 6: Multiply 5 x 2, subtracting the product 10 from the 10 in the dividend, resting your finger on the rod where the 0 was subtracted.

Step 7: Multiply 5 x 0, moving your finger one rod to the right to represent setting this 0.

Step 8: You will note that the last zero in the original dividend of 7,000 was set on the rod to the right of your finger, so it is necessary to add a 0 after the 5 in the quotient, giving you the final quotient of 350.

NOTE: You might prefer setting the dividend, when it ends in zeros, in such a position that the last zero will fall on the last rod to the right

of the abacus. This would mean, perhaps, skipping more than four rods between the divisor and dividend, but it might simplify keeping zeros straight. In the last example, for instance, (7,000 ÷ 20 = 350), you would have counted four rods over from the right end of the abacus setting your dividend, starting on that rod.

LONG DIVISION WITH THREE OR MORE DIGITS IN THE DIVISOR

There is actually no difference between division problems with two digits in the divisor than those with three or more digits. The rule to follow is that all the digits in the divisor are multiplied by each digit in the quotient.

Example 15---95,048 ÷ 436 = 218.

Step 1: Set the problem.

Step 2: Divide 4 into 9, setting the quotient 2 on the second rod to the left; rest your finger to the right.

Step 3: Multiply 2 x 4, subtracting the product 8 from the 9 in the dividend. Rest your finger on that rod.

Step 4: Multiply 2 x 3, subtracting the product 6 from the 15 in the dividend, resting your finger on the rod below the 9.

Step 5: Multiply 2 x 6, subtracting the product 12 from the 90 in the dividend.

Step 6: Divide 4 into 7, setting the quotient 1 on the second rod to the left. Rest your finger to the right.

Step 7: Multiply 1 x 4, subtracting the product 4 from the 7 in the dividend, resting your finger on that rod.

Step 8: Multiply 1 x 3, subtracting the product 3 from the 8 in the dividend, resting your finger on that rod.

Step 9: Multiply 1 x 6, subtracting the product 6 from the rod on which the 4 is set.

Step 10: Divide 4 into 34, setting the quotient 8 immediately to the left. Rest your finger to the right.

Step 11: Multiply 8 x 4, subtracting the product 32 from the 34 in the dividend, resting your finger under the 2.

Step 12: Multiply 8 x 3, subtracting the product 24 from the 28 in the dividend. Rest your finger below the 4.

Step 13: Multiply 8 x 6, subtracting the product 48 from the 48 in the dividend, leaving the final quotient of 218.

<u>Example 16</u>---10,152 ÷ 846 = 12.

Step 1: Set the problem.

Step 2: Divide 8 into 10, setting the quotient 1 immediately to the left, resting your finger to the right.

Step 3: Multiply 1 x 8, subtracting the product 8 from the 10 in the dividend, resting your finger below the 2.

Step 4: Multiply 1 x 4, subtracting the product 4 from the 21 in the dividend, resting your finger on the rod below the 7.

Step 5: Multiply 1 x 6, subtracting the product 6 from the rod to the right of your finger, leaving a new dividend of 1,692.

Step 6: Divide 8 into 16, setting the quotient 2 immediately to the left, resting your finger to the right.

Step 7: Multiply 2 x 8, subtracting the product 16 from the 16 in the dividend. Rest your finger on that rod.

Step 8: Multiply 2 x 4, subtracting the product 8 from the 9 in the dividend, resting your finger on that rod.

Step 9: Multiply 2 x 6, subtracting the product 12 from the 12 in the dividend, leaving the final quotient of 12.

NOTE: In the event the first two digits of the divisor and the dividend are the same, compare the third digits. If the third digit in the divisor is the same or smaller than the third digit in the dividend, set the quotient 1 on the second rod to the left and multiply. If the third digit of the divisor is larger than the third digit of the dividend, set the quotient 9 immediately to the left of the dividend and multiply. In the latter process, your correct quotient will almost always be 9.

The following practice problems have been provided to offer more practice in long division. Also, use the division problems provided in the back of the book.

Exercise I:

1. 408 ÷ 34 = 12
2. 1,035 ÷ 23 = 45
3. 1,904 ÷ 34 = 56
4. 1,296 ÷ 18 = 72
5. 3,196 ÷ 47 = 68
6. 15,129 ÷ 123 = 123
7. 14,061 ÷ 327 = 43
8. 372,922 ÷ 46 = 8,107
9. 37,278 ÷ 654 = 57
10. 14,868 ÷ 63 = 236

Exercise II:

1. 56,088 ÷ 123 = 456
2. 266,424 ÷ 24 = 11,101
3. 9,231 ÷ 17 = 543
4. 9,821 ÷ 61 = 161
5. 1,828,875 ÷ 375 = 4,877
6. 21,472 ÷ 32 = 671
7. 333,315 ÷ 27 = 12,345
8. 14,202 ÷ 18 = 789
9. 163,328 ÷ 29 = 5,632
10. 42,048 ÷ 48 = 876

REVISION INVOLVING MORE THAN ONE DIGIT OF THE DIVISOR

NOTE: This example demonstrates division when more than one digit of the divisor is involved in the revision.

Example 17---16,421 ÷ 2,346 = 6 r2345.

Step 1: Set the problem.

Step 2: Divide the 2 of the divisor into the 16 of the dividend, setting the quotient 8 immediately to the left of the dividend.

Step 3: Multiply 8 x 2, subtracting the product 16 from the 16 in the dividend, resting your finger below the zero.

Step 4: Multiply 8 x 3, and since you can not subtract the product 24 from the rod on which your finger rests and the rod to the right, it is necessary to revise.

Step 5: Subtract 1 from the quotient 8, resting your finger on the rod to the right.

Step 6: Multiply the 1 you subtracted times the 2 of the divisor, setting the product 2 on the rod to the right of your finger. Rest your finger on that rod.

Step 7: Multiply the new quotient 7 x 3 of the divisor, subtracting the product 21 from the rod on which your finger rests and the rod to the right. Rest your finger on that rod.

Step 8: Multiply 7 x 4, subtracting the product 28 from the rod on which your finger rests and the rod to the right. Rest your finger on that rod.

Step 9: Multiply 7 x 6 and you can not subtract the product 42 from the 41 remaining in the dividend, so it is necessary to revise again.

Step 10: Subtract 1 from the quotient 7, resting your finger to the right.

Step 11: Multiply the 1 you subtracted times the 234 of the divisor, adding the product 234 on the three rods to the right of your finger. Rest your finger on the rod on which the 8 is set.

Step 12: Multiply the new quotient 6 times the 6 of the divisor, subtracting the product 36 from the 81 of the dividend, leaving the final quotient of 6 with a remainder of 2345.

Work the following three problems which involve revision where more than one digit of the divisor is included. If you have difficulty with these problems, review the previous example carefully.

1. 4937 ÷ 823 = 5 r822.
2. 28,451 ÷ 347 = 81 r344.
3. 369,056 ÷ 3927 = 93 r3845.

REMAINDERS IN LONG DIVISION

In the previous examples, it was easy to determine the remainders. In each case, only one blank rod remained between the quotient and the remainder. It is sometimes confusing, however, as to whether or not zeros should be added to the quotient, so the following rules and examples will be presented to help eliminate such confusion.

RULE 1: It is necessary to have at least one blank rod between the quotient and the remainder.

RULE 2: Place your finger on the rod where the last digit of the remainder is located; move your finger one rod to the left for each digit in the divisor. Your finger now separates the quotient from the remainder, i.e., numbers to the left of your finger represent the quotient, numbers to the right represent the remainder.

Example 18---5,200,009 ÷ 26 = 200,000 r9.

Step 1: Set the problem.

Step 2: Divide 2 into 5, setting the quotient 2 on the second rod to the left of the dividend.

Step 3: Multiply 2 x 26 of the divisor, subtracting the product 52 from the 52 in the dividend.

Step 4: Since it is impossible to divide 26 into the remaining 9 of the dividend, it is necessary to determine how many zeros must be placed in the quotient.

Step 5: Place your finger on the rod where the remainder 9 is located, move it one rod to the left for each of the two digits in the divisor. The

number to the left of your finger represents the quotient 200,000, while the number to the right represents the remainder 9.

Example 19---4,538,526 ÷ 732 = 6,200 r126.

Step 1: Set the problem.

Step 2: Divide 7 into 45, setting the quotient 6 on the proper rod.

Step 3: Multiply 6 x 7, subtracting the product 42 from the 45 of the dividend.

Step 4: Continue multiplying 6 times the remaining 32 of the divisor, subtracting the product 192 from the dividend, leaving a new dividend of 146,526.

Step 5: Divide 7 into 14, setting the quotient 2 on the proper rod.

Step 6: Multiply 2 times the divisor 732, subtracting the product 1,464 from the dividend, leaving the remaining dividend of 126.

Step 7: Since it is impossible to divide 732 into the remaining 126 of the dividend, it is necessary to determine how many zeros must be placed in the quotient.

Step 8: Place your finger on the rod where the last digit of the remainder 126 is located, move it one rod to the left for each of the three digits in the divisor. (NOTE: See Rule 2) The numbers to the left of your finger represent the quotient--6,200, while the numbers to the right represent the remainder--126.

Work the following three problems and if you have difficulty with them, refer to the previous two examples.

1. 19,680,017 ÷ 82 = 240,000 r17.

2. 10,431 ÷ 745 = 14 r1.

3. 167,405 ÷ 62 = 2,700 r5.

SQUARE ROOTS

It is not difficult to square a number since it is only necessary to multiply that number times itself. For instance, you square the number six by multiplying it by six, giving you the answer of thirty-six. To square twenty-five you multiply by twenty-five, giving the answer of six hundred twenty-five. But to determine the square root of a number, it is necessary to find a particular number which when multiplied by itself will give you the number with which you started. For example, the square root of nine is three (3 x 3 = 9). The square root of four is two (2 x 2 = 4). The square root of sixty-four is eight (8 x 8 = 64).

For those people who want to square a number or find the square root of a number, it is possible to do so on the abacus. It is my recommendation, however, that if an individual is involved in a situation where square roots are used regularly, the best resource to be used would be a logarithm table, which is available in both print and braille.

For those individuals who do wish to find or extract square roots of numbers using the abacus, the examples in this section should provide the necessary training and practice to meet all his needs in this area.

To square a number, the process of multiplication is involved. To extract a square root of a number, however, the process of division is involved. Most of the rules used in regular division can be applied in extracting square roots, though some modification of rules is necessary. These modifications will be discussed as they occur in the examples. It is necessary to have a thorough knowledge of the process of division using the abacus before attempting to master this section.

In regular division problems, your divisor is always stated. In extracting square roots of a number, however, the divisor and the quotient are always the same. It is necessary, therefore, to determine the divisor and quotient at the same time. As in regular division you compare the first number of the divisor to the first number of the dividend, or the first two numbers of the dividend to determine the quotient and its position. In extracting square roots, positioning the quotient is basically the same as that of regular division. If for instance, the first number of the divisor goes into the first number of the dividend, then you skip a rod to set your quotient. But if the first number of the divisor has to be divided into the first two numbers of the dividend, then you place your quotient immediately to the left of the dividend without skipping a rod.

If the dividend contains an odd number of digits, it is necessary to divide into the first digit only. If the dividend contains an even number of digits, however, it is necessary to divide into the first two digits.

Example 1---The square root of 25 is 5.

NOTE: In extracting square roots, set all the problems on the left end of the abacus, skipping the first three rods.

Step 1: Set the number 25 on the abacus starting with the fourth rod from the left end.

Step 2: Determine if the dividend contains an even or odd number of digits--in this case, even.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the dividend 25--in this case, 5.

Step 4: Divide 5 into 25, setting the quotient 5 immediately to the left of the dividend.

NOTE: This is where the quotient would be set in the regular division. Where you are going to have a quotient containing more than one digit, it is necessary to skip a rod if the first number of that quotient is 5 or larger. This will be explained later.

Step 5: Multiply the quotient 5 times itself, subtracting the product 25 from the 25 in the dividend.

NOTE: In this example, you see that your divisor becomes your quotient or vice versa and that it is not necessary to actually set a separate divisor on the abacus.

Example 2---The square root of 64 is 8.

Step 1: Set the dividend 64 on the abacus starting on the fourth rod from the left.

Step 2: Determine if the dividend contains an even or odd number of digits--in this case, even.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the dividend 64--in this case, 8.

Step 4: Divide 8 into 64, setting the quotient 8 immediately to the left of the dividend.

Step 5: Multiply 8 times itself, subtracting the product 64 from the 64 in the dividend.

Example 3---The square root of 625 is 25.

Step 1: Set the dividend 625 on the abacus, again beginning on the fourth rod from the left.

Step 2: Determine if the dividend contains an even or odd number of digits--in this case, odd.

NOTE: Since the dividend contains an odd number of digits, it is necessary to divide into the first digit only. It is also necessary to skip

a rod to the left of the dividend in setting the quotient.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the first number of the dividend 6--in this case, 2.

Step 4: Divide 2 into the 6 of the dividend, setting the quotient 2 on the second rod to the left of the dividend.

NOTE: Since the divisor and the quotient are always the same in extracting square roots, it is necessary to say that 2 will go into 6 two times. This gives you the number 2 as the divisor as well as the quotient.

Step 5: Multiply 2 times itself, subtracting the product 4 from the 6 in the dividend, leaving a new dividend of 225.

Step 6: Double the quotient 2, leaving it a 4.

NOTE: It is necessary to double each digit in the quotient after it has been used as a multiplier.

Step 7: Divide the quotient 4 into the 22 of the dividend, setting the quotient 5 immediately to the left of the dividend.

Step 8: Multiply the new quotient 5 times the old quotient 4, subtracting the product 20 from the 22 of the dividend.

Step 9: Multiply 5 times itself, subtracting the product 25 from the remaining 25 of the dividend.

Step 10: Double the quotient 5, leaving a total of 50 on the abacus.

Step 11: Divide the final quotient 50 by 2, giving the final correct answer of 25.

NOTE: As stated above, it is necessary to double each digit in the quotient after it has been used as a multipler. It is then necessary to divide the total quotient by 2 in order to determine the final correct answer. In the event that this can not be done by mental calculation, it will be necessary to set the problem up as a regular division problem, with

the 2 as your divisor and the total quotient as your dividend, i.e., 50 ÷ 2 = 25.

Example 4---The square root of 1156 is 34.

Step 1: Set the problem.

Step 2: Determine if the dividend contains an even or odd number of digits--in this case, even.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the first two numbers of the dividend 11--in this case, 3.

Step 4: Divide 3 into 11, setting the quotient 3 immediately to the left of the dividend.

Step 5: Multiply 3 times itself, subtracting the quotient 9 from the 11 of the dividend, leaving a new dividend of 256.

Step 6: Double the quotient 3, making it a 6.

Step 7: Divide the new quotient 6 into the 25 of the dividend, setting the quotient 4 immediately to the left of the dividend.

Step 8: Multiply 4 times 64, subtracting the product 256 from the 256 in the dividend.

Step 9: Double the 4 of the quotient, making it the total quotient of 68.

Step 10: Divide 68 by 2, giving the final correct answer of 34.

Example 5---The square root of 729 is 27.

Step 1: Set the problem.

Step 2: Determine that the dividend contains an odd number of digits.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the first number of the dividend 7--in this case, 2.

Step 4: Divide 2 into 7, setting the quotient 2 on the second rod to the left of the dividend.

Step 5: Multiply 2 times itself, subtracting the product 4 from the 7 in the dividend.

Step 6: Double the quotient 2, making it a 4.

Step 7: Divide 4 into the 32 of the dividend, setting the quotient 8 immediately to the left.

Step 8: Multiply 8 times 4, subtracting the product 32 from the 32 of the dividend.

Step 9: Multiply 8 times itself and it is impossible to subtract the product 64 from 09 in the dividend. It is necessary therefore to revise the quotient 8.

Step 10: Subtract 1 from the quotient 8, leaving the revised number 7.

Step 11: Multiply the 1 which was subtracted from the quotient 8 times the 4 of the quotient, adding the number 4 back into the dividend.

Step 12: Multiply 7 times itself, subtracting the product 49 from the 49 of the dividend.

Step 13: Double the quotient 7 showing the total quotient of 54.

Step 14: Divide 54 by 2, giving the final correct answer of 27.

Example 6---The square root of 3969 is 63.

Step 1: Set the problem.

Step 2: Determine that it has an even number of digits.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the first two numbers of the dividend 39--in this case, 6.

Step 4: Divide 6 into 39, setting the quotient 6 on the second rod to the left.

NOTE: Any time the first number of your quotient is 5 or larger, it is necessary to skip a rod before setting the number.

Step 5: Multiply 6 times itself, subtracting the product 36 from the 39 in the dividend, leaving a dividend of 369.

Step 6: Double the quotient 6, making it 12.

Step 7: Divide 1 into the 3 of the dividend, setting the quotient 3 on the second rod to the left.

Step 8: Multiply 3 times the total quotient of 123, following the rules of division.

Step 9: Double the quotient 3, giving the total quotient of 126.

Step 10: Divide 126 by 2, giving the final correct answer of 63.

Example 7---The square root of 501,264 is 708.

Step 1: Set the problem.

Step 2: Determine that there are an even number of digits.

Step 3: Think of the nearest number which when multiplied by itself will not exceed the first two numbers of the dividend 50--in this case, 7.

Step 4: Divide 7 into 50, setting the quotient 7 on the second rod to the left of the dividend.

Step 5: Multiply 7 times itself, subtracting the product 49 from the 50 in the dividend, leaving a new dividend of 11,264.

Step 6: Double the 7, making it a 14.

Step 7: Following the regular rules of division, it is impossible to compare the 1 of the divisor to the 1 of the dividend, so it is necessary to compare the second number of the divisor to the second number of the dividend. Since 4 of the divisor is larger than 1 of the dividend, it is necessary to use the rule of the nines, and set a nine to the left of the dividend without skipping a rod.

Step 8: Multiply 9 times 1, subtracting the product 9 from the 11 of the divisor.

Step 9: Multiply 9 times 4 and it is impossible to subtract the product 36 from the 22 of the dividend, so it is necessary to revise the quotient 9 to the quotient 8.

Step 10: Through revision, add the number 1 back into the dividend, making it read 3264.

Step 11: Multiply 8 times the quotient 408, subtracting the product 3264 from the remaining 3264 in the dividend.

Step 12: Double the quotient 8, giving 1416.

Step 13: Divide 1416 by 2, giving the final correct answer of 708.

Example 8---The square root of 2 is 1.41

Step 1: Set the problem.

Step 2: Think of the nearest number which when multiplied by itself will not exceed the first number of the dividend 1--in this case, 1.

Step 3: Divide 1 into 2, setting the quotient 1 on the second rod to the left of the dividend.

Step 4: Multiply 1 times itself, subtracting the product 1 from the 2 of the dividend, leaving the new dividend of 1.

NOTE: It is necessary to add zeros to the dividend along with adding a decimal point. You can continue adding zeros to extend the answer beyond the decimal point as many places as are desired.

Step 5: Double the quotient 1, making it read 2.

Step 6: Divide 2 into 10, setting the quotient 5 immediately to the left of the dividend.

Step 7: Multiply 5 times 2, subtracting the product 10 from the 10 in the dividend.

Step 8: In trying to multiply 5 times itself there is no dividend from which to subtract the product 25. It is necessary therefore to revise

our quotient 5.

Step 9: Revise the quotient 5 to 4 (as in regular division), replacing the number 2 back into the dividend.

Step 10: Multiply 4 times itself, subtracting the product 16 from the 20 in the dividend, leaving 4.

Step 11: Double the 4 of the quotient, making a total of 28.

Step 12: Add zeros to the dividend.

Step 13: Divide the 2 of the divisor into the 4 of the dividend, setting the quotient 2 on the second rod to the left of the dividend.

Step 14: Multiply the new quotient 2 times the first two of the quotient and subtract the product 4 from the dividend, clearing the abacus.

Step 15: Add another zero and multiply 1 times the remaining 81 of the quotient, subtracting the product 81 from the dividend, leaving a remainder of 119. This remainder can now be discarded. It is possible, however, to continue to add zeros to the dividend so that the answer can be taken out as many decimal places as is desired.

Step 16: Double the 1 of the quotient, giving a total quotient of 2.82.

Step 17: Divide 2.82 by 2, giving the final correct total of 1.41.

NOTE: It is necessary to remember where zeros are added to dividends in order to determine where the decimal point should go in the quotient.

Dividends which end in zero can be placed so that the last digit can terminate on the last rod of the abacus. This is the method used in regular long division. Using this method, however, does not enable the abacus operator to add additional zeros in order to extend the answer beyond a decimal point. In my experience, it is simplest to remember on which rod the dividend ends when it ends in a zero and to remember that it is necessary to add a decimal point in your answer if you go beyond that point by adding zeros to

your dividend.

Exercise I:

The square root of ____ is ____.

1. 484 = 22.
2. 196 = 14.
3. 1369 = 37.
4. 3025 = 55.
5. 8464 = 92.
6. 5625 = 75.
7. 26,569 = 163.
8. 848,241 = 921.
9. 16,016,004 = 4002.
10. 494,209 = 703.

Exercise II:

The square root of ____ is ____.

1. 35 = 5.91
2. 46 = 6.78
3. 40 = 6.32
4. 133 = 11.53
5. 125 = 11.18
6. 273 = 16.52
7. 469 = 21.65
8. 560 = 23.66
9. 926 = 30.43
10. 1000 = 31.62

SQUARE ROOTS OF DECIMALS

Extracting square roots of decimals on the abacus takes more mental calculation than extracting the square roots of whole numbers. It is possible to set your dividend in a way that the quotient will fall so that its decimal point is in the proper place on the abacus. As shown in the previous examples, it is necessary to divide the original quotient by 2 in order to get the final correct quotient, which means that it is then necessary to reposition your original quotient (which now becomes a dividend) in such a way that when divided by 2 your new quotient's decimal point falls in the proper place on the abacus. It is therefore simplest to remember how many decimal points are needed in the final quotient.

In multiplication of decimals, you have as many decimal points in your product as there are in the multiplier and multiplicand combined, thus in the division of decimals, you have half as many decimal points in your quotient as you have in your dividend. Too, in extracting square roots of decimals you must have an even number of digits in the dividend. Since this is true it is sometimes necessary to add zeros, and these must be added to the end of the dividend and not at the beginning. So, .012 would have to become .0120.

Decimals that do not begin with zeros can be treated as a regular whole number, remembering to add a zero if necessary to make it an even number, and to point off half as many decimal places in the final quotient as there were in the dividend. After determining that the dividend has an even number of digits, and if it begins with one or more zeros, it is best to drop the zeros considering the remainder of the dividend as a whole

number. It is then necessary to add enough zeros back into the quotient so that it would have the proper number of decimal places in the quotient.

Example 1---The square root of .0025 is .05

Step 1: Determine that it has an even number of digits.

Step 2: Drop the two zeros, leaving a dividend of 25.

Step 3: Set the problem on the proper rods starting with the fourth rod from the left.

Step 4: Think of the nearest number which when multiplied by itself will not exceed the first two numbers of the dividend 25--in this case, 5, setting the 5 immediately to the left of the dividend.

Step 5: Multiply 5 times itself, subtracting the product 25 from the 25 in the dividend.

NOTE: Since the original dividend of .0025 contained four decimal places, it is necessary to have two decimal places in the answer. It is necessary therefore to add a zero back into the answer, giving the final correct quotient of .05.

As stated before, in order to be sure that the quotient has half as many decimal places as the dividend, it is necessary at times to add zeros. These zeros must be added immediately following the decimal point.

Example 2---The square root of .0064 is .08

Step 1: Determine that it has an even number of digits.

Step 2: Drop the two zeros, leaving a dividend of 64.

Step 3: Set the problem on the proper rods starting with the fourth rod from the left.

Step 4: Think of the nearest number which when multiplied by itself will not exceed the first two numbers of the dividend 64--in this case, 8.

Step 5: Multiply 8 times itself, subtracting the product 64 from the 64 in the dividend.

NOTE: Since there were four decimal places in the original dividend of .0064, the quotient must have two decimal places. It is necessary then to add a zero before the quotient 8, giving a final correct quotient of .08.

Example 3---The square root of .002 is .044

Step 1: Add a zero, making the dividend an even number--.0020.

Step 2: Drop the first two zeros, leaving a dividend of 20.

Step 3: Set the problem on the proper rods starting with the fourth rod from the left.

Step 4: Think of the nearest number which when multiplied by itself will not exceed the dividend 20--in this case, 4, setting the 4 immediately to the left of the dividend.

Step 5: Multiply 4 times itself, subtracting the product 16 from the 20 in the dividend, resting your finger on the second rod.

Step 6: Double the quotient 4, making it an 8.

Step 7: Add a zero to the dividend 4, making it 40.

NOTE: It will be necessary to add an additional decimal place in the answer.

Step 8: Divide 8 into 40, setting the quotient 5 immediately to the left of the dividend.

Step 9: Multiply 5 times 8, subtracting the product 40 from the 40 in the dividend.

Step 10: Since you can not multiply 5 x 5 subtracting the product 25 from the dividend, it is necessary to revise the quotient to a 4, adding the number 8 back into the dividend to the right of your finger and adding a zero.

Step 11: Multiply 4 x 4, subtracting the product 16 from the 80 in the dividend.

Step 12: Double the 4 in the quotient, making a total quotient of 88.

Step 13: Divide the quotient 88 by 2, giving the quotient of 44.

NOTE: To the original dividend of .002, it was necessary to add a zero to make it an even number. In our computation, it was necessary to add two additional zeros, making the total dividend a six-digit number. Since it is necessary to have half as many decimal places in the quotient as there are in the dividend, it is necessary to add a zero in the final quotient of .44, giving the final correct answer of .044.

Exercise:

The square root of ____ is ____.

1. .0036 = .06
2. .001369 = .037
3. .00000784 = .0028
4. .0529 = .23
5. .000049 = .007

DECIMALS

In working problems with decimals, it is necessary to utilize the unit markers on the bar of the abacus. As stated earlier in working problems with money, simply let the last two rods to the right of the abacus represent the "cents" rods. It is not always possible to do this with other problems containing decimals since there might be more than two rods needed after the decimal point.

In addition and subtraction of decimals, simply follow the rules of regular addition and subtraction. It might be necessary to move farther to the left on the abacus in order to keep the unit marker, "decimal point", in the proper place. The following examples will illustrate this method.

<u>Example 1</u>---6.134 + .282 = 6.416

Step 1: Set the problem on the right end of the abacus, using the unit marker, i.e., to the left of the third rod, as the decimal point.

Step 2: Add, using complementary numbers as in all addition.

<u>Example 2</u>---2.15 + 7.02 = 9.17

Step 1: Set 2.15 so that the .15 is set on the two rods immediately to the right of the unit marker. Add the 7.02 in such a way that the .02 falls on the same two rods as the .15.

NOTE: In this problem, you did not use the first rod to the right, but this will not cause confusion. Zeros on the end of a decimal do not change its value.

<u>Example 3</u>---.6237 + .98 = 1.6037

Step 1: Set .6237 immediately to the right of the unit marker, i.e., between the sixth and seventh rods from the right.

NOTE: In order to use a unit marker, it will be necessary to set the problem farther to the left on the abacus, using the second unit marker as the decimal point.

Step 2: Add .98 immediately to the right of the same marker, giving a sum of 1.6037.

NOTE: Some students have no problem remembering where the decimal point falls and do not use the unit marker. It is not difficult to use, however, and its use should be encouraged to avoid careless mistakes.

Exercise:

1. 1.955 + 3.12 + .573 + 7 + 4.8 = 17.448
2. 23.12 + 9.88 + 71.9 + .14 + 5.8 = 110.84
3. 5.743 + 22.35 + .875 + 31.13 + 12 = 72.098
4. 2.987 + 71.8 + 12.47 + .367 + 4.531 = 92.155
5. 98.251 + .642 + .4 + 2.3 + 7.35 = 108.943

DECIMALS

Multiplication

In multiplication of decimals, you follow the same rules of regular multiplication, except you do not necessarily always skip two rods between the multiplicand and multiplier. After setting your number to the left, set the multiplier in such a way that your final product ends the proper number of rods to the right of a unit marker. It was explained in the section on multiplication how to locate the rod on which the last digit of the product would be located. Continue to use this method here, except you should make this determination before setting the multiplier. Examples will be used to demonstrate this procedure.

NOTE: In multiplication of decimals, you have as many digits after the decimal point in your product as you have in your multiplier and multiplicand combined. For instance, in a problem where there are two digits following the decimal point in the multiplier, and three digits following the decimal point in the multiplicand, then the final product will have a total of five digits following the decimal point.

Example 1---3.2 x 4.18 = 13.376

Step 1: Set 3.2 to the left.

Step 2: Before setting the multiplier to the right, determine where you want the product to end on the abacus. Remember to allow one rod for each digit in the multiplicand, plus one additional rod to the right of the multiplier. Since the final product in this problem has three digits after the decimal point, it is necessary to have the product terminate on the third rod to the right of a unit marker. Since the present problem has

two digits in the multiplicand on the left, the final product will end three rods to the right of the multiplier. Too, since there are three digits after the decimal point, then it is necessary to set the multiplier so that the last digit of it will fall on the rod immediately to the left of the unit marker, i.e., the fourth rod from the right.

Step 3: After resting your finger to the right of the multiplier, begin the multiplication. Multiply 8 x 3, setting the product 24 on the rod where your finger is resting and the rod to the right. Rest your finger on that rod.

Step 4: Multiply 8 x 2, setting the product 16 on the rod where your finger rests and the rod to the right. Clear the multiplier 8. Rest your finger on that rod.

Step 5: Multiply the multiplicand 3.2 by the 1 of the multiplier, setting the product on the two rods to the right of your finger. Clear the one, and rest your finger on that rod.

Step 6: Multiply 4 x 3.2, setting the product on the three rods to the right of the 4, then clear the 4. This leaves the final product of 13.376 on the five rods to the right of the abacus. The decimal point falls in the proper place.

<u>Example 2</u>---1.23 x .46 = .5658

Step 1: Set 1.23 to the left, .46 to the right, in this particular case, skipping two rods.

NOTE: In determining where to set the multiplier, it was necessary to first determine how many places would be pointed off in the product, i.e., 4, the combined number of places in the multiplier and multiplicand. It is necessary, therefore, that the product end four rods to the right of a unit marker. Secondly, since there are three digits in the multiplicand,

the product will automatically end on the fourth rod to the right of the multiplier, so it was necessary to set the multiplier immediately to the left of a unit marker.

Step 3: Multiply the problem using the regular rules of multiplication, and your final product, .5658, will be formed on the rods immediately to the right of the unit marker, the second marker from the right.

NOTE: In order to work longer problems and in order to keep the product on the proper rods, it might be necessary to use a second abacus.

The following exercise will offer practice material for multiplying decimals.

Exercise:

1. 3.5 x 2.3 = 8.05
2. 6.12 x 7.9 = 48.348
3. 23 x 5.35 = 123.05
4. 57.2 x .15 = 8.58
5. 9.64 x 82.4 = 794.336

DECIMALS

Division

In dividing numbers with decimals, it is simplest to make the divisor a whole number by moving the decimal point as many places as necessary to the right, and moving the decimal point in the dividend the same number of places. This will not affect your final quotient. Since you have as many places after the decimal point in your quotient as you do in your dividend, it is easy to set your dividend in a way that the quotient will fall so that its decimal point is in the proper place on the abacus. To determine the proper place to set the dividend, choose a unit marker in the center of the abacus and allow one rod to the right of it for each decimal place in the dividend, as well as an additional rod for each digit in the divisor, plus one additional rod. This rod is the one on which the dividend should terminate.

NOTE: Many people prefer simply to remember the number of decimal points needed in the quotient, since it is always the same as the number of decimal points in the dividend as mentioned above.

Use the following examples to determine this process.

Example 1---4.83 ÷ 23 = .21

Step 1: Set the divisor 23 to the left.

Step 2: Choose the first unit marker to the right of the divisor and allow two rods to the right for the two decimal places in the dividend, .83, plus two additional rods for the two digits in the divisor and one additional rod. The last number of the dividend should be set on this rod. This allows four unused rods to fall between the divisor and the dividend.

Step 3: Divide 23 into 4.83 and the final quotient, .21, will fall on the two rods immediately to the right of the unit marker.

Example 2---4.0334 ÷ 67 = .0602

Step 1: Set 67 to the left.

Step 2: Choose the unit marker to the right of the divisor, count over to the seventh rod to the right, i.e., four for the decimal places plus two for the divisor and one additional rod. The dividend should terminate on this rod. This allows four unused rods to fall between the divisor and the dividend.

Step 3: Divide 67 into 4.0334, and the final quotient is .0602.

Exercise:

1. 24.24 ÷ 12 = 2.02
2. 76.3 ÷ 9.1 = 8.38
3. 186.52 ÷ 27 = 6.908
4. 54.201 ÷ 87 = .623
5. 1001.2 ÷ 418 = 2.4

FRACTIONS

It is possible to handle fractions on the abacus, though it is sometimes necessary to either write down some of the facts involved or keep them in your head. I have found that students have not, in general, been interested in learning to work fractions on the abacus; but for those who are interested, the following information should make it possible for them to handle all their needs in this area.

In computing fractions with pencil and paper or in braille, it is mostly a process of either multiplication or division. It is not difficult to complete these operations on the abacus, but the most difficult part seems to be in setting fractions which are not possible in the usual sense of the word, i.e., placing the numerator over the denominator. This necessitates either keeping the information in your head or writing it down. Simple fractions, however, can be set on the abacus by simply setting the numerator on the left end while setting the denominator on the right, or by skipping one rod between the numerator and denominator. Again, it is necessary to keep these facts in your head.

The following examples will be used to demonstrate how to handle fractions on the abacus.

Example 1---2/3 + 3/5 = 1 4/15.

Step 1: Set 2/3 on the left by setting 2 on the first rod and setting 3 to the right, skipping one rod. The rod you skip is the imaginary fraction line.

Step 2: Set 3/5 on the right end of the abacus using the same method.

Step 3: It is necessary to use a common denominator, 15, in order to add this type of fraction where you have different denominators. Set the common denominator 15 in the center of the abacus.

Step 4: Mentally divide the 3 on the left into 15, and multiply your quotient 5 x 2. Set the product 10 on the left after clearing the fraction 2/3.

Step 5: Mentally divide the 5 on the right into 15, and multiply your quotient 3 x 3. Add the product 9 to the 10 on the left. Clear the 3/5.

NOTE: Now the fraction 19/15 remains on the abacus. It is necessary to reset this fraction so that you can divide 15 into 19 using the regular rules of division. Thus, you arrive at the quotient 1 4/15.

Example 2---3/7 x 2,487 = 1,065 6/7.

NOTE: In this type problem where you have only one fraction to deal with, it is possible to set the numerator on the left end of the abacus and the denominator on the right end.

Step 1: Set the numerator 3 on the first rod to the left and set the denominator 7 on the first rod to the right.

Step 2: Skip two places to the right of the 3 on the left and set the number 2,487.

Step 3: Multiply 3 x 2,487 setting the product 7,461 in its proper place according to the rules of multiplication.

Step 4: Clear the 3 on the left, replacing it with the denominator 7 on the right.

Step 5: Divide 7 into 7,461, leaving the quotient 1,065 6/7.

NOTE: The examples used so far have dealt with simple fractions. In the case of mixed numbers where you have whole numbers and fractions, it is

necessary to record each step of the operation either in pencil or braille, or perhaps a second abacus. As stated earlier in this text, the abacus will show only the final answers, and work leading to these answers must sometimes be recorded elsewhere.

The following exercises can be completed on the abacus by following the rules of computing fractions and by following the rules of the abacus regarding addition, subtraction, multiplication and division.

Exercise I:

1. 4/5 + 1/4 = 1 1/20
2. 8 1/7 + 3/4 = 8 6/7
3. 10 1/2 + 15 3/4 = 26 1/4
4. 3/8 + 5 2/3 = 6 1/24
5. 5/9 + 2/7 = 53/63

Exercise II:

1. 7/9 x 4/5 = 28/45
2. 3 1/6 x 8/9 = 2 22/27
3. 14 x 13 5/16 = 186 3/8
4. 72 1/9 x 3/17 = 12 111/153
5. 3 5/16 x 18 32/33 = 62 221/264

NEGATIVE NUMBERS

When working with negative numbers, continue to use the principle of complementary numbers. The following examples will demonstrate the addition and subtraction of negative numbers.

Example 1---28 - 47 = -19.

Step 1: Set the +28 on the left end of the abacus, so it will not be forgotten, and set the -47 on the right end of the abacus.

Step 2: Since you can not subtract 47 from 28 directly, it is necessary to subtract the smaller number, i.e., 28 from the larger number 47.

Step 3: Subtract 28 from the 47, leaving the -19.

Example 2---237 - 488 = 251.

Step 1: Set 237 on the left of the abacus and 488 on the right.

Step 2: Subtract 237 from 488, leaving the -251.

Example 3---23 + -46 = -23.

Step 1: Set the +23 on the left and set -46 on the right.

Step 2: Subtract 23 from 46, leaving the -23.

Example 4---45 + -87 = -42.

Step 1: Set +45 on the left and set -87 on the right.

Step 2: Subtract 45 from -87, leaving -42.

NOTE: By always setting the plus numbers on the left and the minus numbers on the right, you will avoid getting them confused.

BOWLING

Bowling is a sport which has become very popular amongst blind people; and though a game always has a scorekeeper, some blind bowlers have indicated a desire to be able to keep their own score. The abacus offers this opportunity. It is assumed here that the abacus operator already has sufficient knowledge of the method by which bowling games are scored. In the event this is not true, it would be wise to first seek this information from a bowling friend before attempting this chapter.

Keeping bowling scores on the abacus is simple and easy, though it is somewhat difficult to explain. Only one person's score should be kept on each abacus.

Starting from the left end of the abacus, use the first rod to indicate the frames which have been totaled. The third rod is used to indicate strikes and spares, or what might be called untotaled frames. The lower beads on this rod indicate strikes, while the five-bead represents a spare. The fifth rod indicates the score of an uncompleted frame. The total score is kept on the right end of the abacus where addition is normally done.

A practice game will perhaps best demonstrate this method.

First ball, first frame -- you score seven pins, and this is set on the fifth rod from the left on the abacus.

Second ball, first frame -- you pick up a spare, so set the five-bead on the third rod from the left and clear the seven showing on the fifth rod. The five-bead on the third rod indicates that you have spared in the first frame.

First ball, second frame -- you score six pins, so set this number on the fifth rod. It is now possible to total the first frame, which is

sixteen. Set one on the first rod left to indicate the first frame is being totaled, clear the spare bead on the third rod, and set the total, sixteen, on the right end of the abacus. The abacus now shows one frame totaled with six in the uncompleted second frame.

Second ball, second frame -- you pick up three, so you can now total the second frame since you did not strike or spare that frame. Set a second bead on the first rod, clear the six off the fifth rod, and add the total of nine to the total on the right. The abacus now shows two frames totaled with twenty-five pins.

First ball, third frame -- is a strike, so set one on the third rod.

First ball, fourth frame -- is another strike, so set a second bead on the third rod. The abacus now shows two frames totaled, with a strike in the third and fourth frames.

First ball, fifth frame -- is a five, and this number is set on the fifth rod. It is now possible to total the third frame. Add one to the first rod, clear one from the third rod, and add the total of the third frame, twenty-five, to the total on the right.

The abacus now shows three frames totaled for fifty pins, a strike in the fourth frame, and a five in the incomplete fifth frame.

Second ball, fifth frame -- scores four, so it is possible to now total the fourth and fifth frames. Add one to the first rod left, clear the strike bead on the third rod, and add the total of the fourth frame, nineteen, to the total on the right. Now add one on the first rod, clear the nine on the fifth rod, adding it to the total on the right. This nine is the total of the fifth frame.

The abacus now shows five frames totaled for seventy-eight pins.

First ball, sixth frame -- is an eight, so set this number on the fifth rod.

Second ball, sixth frame -- picks up a spare, so set the five-bead on the third rod, and clear the eight on the fifth.

First ball, seventh frame -- is a strike, so set one on the third rod. You can now total the sixth frame, so add one on the first rod, clear the five-bead on the third, and add twenty, the score of the sixth frame, to the total on the right.

The abacus now shows six frames totaled for a total of ninety-eight pins, and a strike in the seventh frame.

First ball, eighth frame -- is six, so set that number on the fifth rod.

Second ball, eighth frame -- picks up three more pins, so add that number to the fifth rod, totaling nine on that rod. It is now possible to total the seventh frame, so add one on the first rod, clear the one on the third rod, and add nineteen, the total of the seventh frame, to the total on the right. It is also possible to total the eighth frame which is nine, so set one on the first rod, clear the nine on the fifth rod, adding it to the total on the right. The abacus now shows eight frames totaled for one hundred and twenty-six pins.

Ninth and tenth frames -- assuming you strike both these frames, set two ones on the third rod.

First ball after the tenth frame -- scores seven, so set seven on the fifth rod. Now total the ninth frame by adding one on the first rod, clear one from the third rod, and add the total of that frame, twenty-seven, to the total on the right.

With your last ball, you pick up your spare, giving you a total in the tenth frame of twenty, and this is added to the total on the right, clearing all the beads on the left end of the abacus. You have a final

score of one seventy-three, and that is not a bad game.

This example has shown the different possibilities you will be faced with in scorekeeping, and if necessary carefully review this chapter.

Appendix 1

ADDITION

Exercise I:

1. 5 + 6 + 6 + 2 + 3 + 4 + 5 + 1 + 4 + 2 = 38
2. 5 + 5 + 4 + 6 + 1 + 6 + 2 + 5 + 4 + 3 + 3 = 44
3. 1 + 6 + 4 + 3 + 6 + 2 + 1 + 8 + 5 + 4 + 1 = 41
4. 7 + 2 + 6 + 9 + 1 + 8 + 3 + 5 + 6 + 3 = 50
5. 8 + 6 + 1 + 4 + 5 + 7 + 9 + 2 + 6 + 3 + 8 + 4 + 3 + 5 + 4 + 6 + 3 + 8 + 9 + 6 + 3 + 2 + 4 + 7 = 123
6. 8 + 6 + 5 + 2 + 4 + 1 + 3 + 7 + 6 + 8 + 1 + 9 + 5 + 3 + 8 + 9 + 7 + 2 + 1 + 9 = 104
7. 7 + 8 + 3 + 1 + 5 + 7 + 2 + 4 + 6 + 8 + 5 + 9 + 7 + 4 + 2 + 8 + 8 + 7 + 5 = 106
8. 6 + 6 + 8 + 3 + 1 + 1 + 4 + 4 + 5 + 9 + 7 + 8 + 6 + 4 + 3 + 8 + 9 + 7 + 5 = 104
9. 6 + 3 + 1 + 4 + 8 + 2 + 5 + 3 + 9 + 7 + 4 + 8 + 5 + 1 + 9 + 7 + 3 + 9 + 4 + 6 = 104
10. 2 + 3 + 5 + 8 + 2 + 4 + 6 + 5 + 9 + 7 = 51

Exercise II:

1. 4 + 7 + 2 + 3 + 5 + 6 + 1 + 9 + 8 + 7 = 52
2. 8 + 2 + 3 + 6 + 4 + 9 + 5 + 7 + 8 + 3 = 55
3. 6 + 8 + 1 + 4 + 6 + 5 + 3 + 9 + 2 = 44
4. 7 + 8 + 2 + 3 + 4 + 5 + 9 + 9 + 6 + 2 = 55
5. 3 + 1 + 7 + 6 + 9 + 2 + 4 + 3 + 5 + 8 = 48
6. 9 + 1 + 7 + 3 + 6 + 4 + 1 + 2 + 3 + 6 = 42
7. 7 + 2 + 9 + 4 + 1 + 3 + 9 + 5 + 8 + 6 = 54
8. 5 + 1 + 7 + 3 + 5 + 8 + 4 + 6 + 2 + 8 = 49
9. 2 + 5 + 7 + 8 + 6 + 9 + 3 + 5 + 1 + 4 = 50
10. 8 + 6 + 2 + 1 + 7 + 9 + 4 + 9 + 7 + 8 = 61

Exercise III:

1. 27 + 19 + 38 + 46 + 57 + 98 + 20 + 14 + 33 + 16 = 368
2. 45 + 22 + 91 + 28 + 37 + 56 + 60 + 11 + 88 + 75 = 513
3. 42 + 18 + 76 + 99 + 53 + 75 + 32 + 81 + 22 + 69 = 567
4. 84 + 27 + 62 + 13 + 75 + 96 + 41 + 99 + 78 + 87 = 662
5. 78 + 52 + 43 + 26 + 94 + 19 + 85 + 37 + 58 + 63 = 555
6. 62 + 85 + 17 + 48 + 66 + 59 + 33 + 95 + 24 + 31 = 520
7. 38 + 27 + 94 + 61 + 72 + 15 + 38 + 40 + 81 + 29 = 495
8. 53 + 34 + 25 + 76 + 39 + 64 + 73 + 59 + 81 + 22 = 526
9. 39 + 11 + 73 + 67 + 96 + 24 + 41 + 32 + 53 + 86 = 522
10. 75 + 81 + 27 + 30 + 45 + 98 + 94 + 62 + 83 + 19 = 614

Exercise IV:

1. 34 + 54 + 12 + 46 + 55 + 25 + 65 + 42 + 36 + 13 = 382
2. 62 + 13 + 42 + 34 + 63 + 25 + 60 + 32 + 46 + 12 = 389
3. 44 + 21 + 13 + 43 + 35 + 62 + 24 + 46 + 60 + 32 = 380
4. 72 + 16 + 55 + 91 + 88 + 27 + 83 + 72 + 69 + 94 = 667
5. 48 + 15 + 99 + 77 + 31 + 24 + 50 + 66 + 18 + 83 + 14 = 525
6. 22 + 96 + 33 + 87 + 45 + 71 + 19 + 55 + 47 + 16 = 491
7. 38 + 27 + 94 + 61 + 72 + 15 + 38 + 40 + 81 + 29 = 495
8. 53 + 34 + 25 + 76 + 39 + 64 + 73 + 59 + 81 + 22 = 526
9. 39 + 11 + 73 + 67 + 96 + 24 + 41 + 32 + 53 + 86 = 522
10. 75 + 81 + 27 + 30 + 45 + 98 + 94 + 62 + 83 + 19 = 614

Exercise V:

1. 178 + 299 + 364 + 755 + 418 + 991 + 337 + 615 + 415 + 256 = 4,628
2. 987 + 321 + 456 + 789 + 123 + 345 + 456 + 772 + 987 + 332 = 5,568
3. 128 + 919 + 317 + 432 + 239 + 336 + 455 + 982 + 274 + 616 = 4,698
4. 917 + 786 + 413 + 418 + 723 + 995 + 547 + 418 + 111 + 619 = 5,947
5. 377 + 462 + 555 + 380 + 919 + 827 + 345 + 628 + 872 + 426 = 5,791
6. 875 + 988 + 346 + 552 + 219 + 367 + 456 + 652 + 189 + 777 = 5,421
7. 852 + 916 + 324 + 217 + 818 + 917 + 329 + 654 + 782 + 616 = 6,425
8. 404 + 697 + 523 + 118 + 923 + 757 + 628 + 937 + 645 + 230 = 5,862
9. 318 + 999 + 782 + 645 + 321 + 118 + 227 + 389 + 645 + 634 = 5,078
10. 376 + 187 + 452 + 369 + 532 + 456 + 761 + 907 + 883 + 536 = 5,459

Exercise VI:

1. 1,182 + 7,516 + 3,321 + 1,987 + 2,346 + 5,555 + 6,191 + 1,782 + 7,155 + 2,133 = 39,168
2. 4,718 + 3,312 + 1,962 + 8,875 + 1,877 + 6,253 + 7,774 + 3,618 + 9,211 + 6,787 = 54,387
3. 9,633 + 1,187 + 3,256 + 6,687 + 9,281 + 6,665 + 5,050 + 3,327 + 2,225 + 3,726 = 51,037
4. 8,857 + 5,860 + 1,041 + 4,854 + 3,594 + 8,369 + 1,845 + 2,831 + 4,654 + 3,347 = 45,252
5. 6,672 + 5,786 + 1,056 + 2,391 + 2,572 + 7,476 + 1,127 + 2,136 + 6,098 + 9,454 = 44,768
6. 1,876 + 1,971 + 2,789 + 6,656 + 1,198 + 7,233 + 2,918 + 3,641 + 8,655 + 3,375 = 40,312
7. 2,122 + 3,187 + 9,196 + 4,572 + 3,138 + 5,157 + 6,228 + 4,774 + 5,277 + 7,544 = 51,195
8. 3,335 + 1,785 + 9,192 + 3,326 + 5,578 + 2,741 + 6,832 + 1,198 + 8,724 + 1,787 = 44,498
9. 9,433 + 2,876 + 1,152 + 3,781 + 3,922 + 2,581 + 7,753 + 6,218 + 6,187 + 7,992 = 51,895
10. 8,177 + 6,152 + 6,881 + 7,521 + 1,111 + 3,378 + 5,642 + 7,718 + 7,529 + 2,336 = 56,445

Exercise VII:

1. 31 + 457 + 218 + 99 + 10 + 57,298 + 44 + 133 + 882 + 29,500 + 187,622 = 276,294
2. 25 + 417 + 98 + 6,442 + 77 + 3,612 + 18 + 9 + 4,298 + 333 + 191 + 75 = 15,595
3. 3,519 + 17 + 42 + 379 + 25 + 1,072 + 25,261 + 144,729 + 35 + 1,032 = 176,111
4. 21 + 468 + 379 + 842 + 15 + 44,298 + 3 + 406 + 1,919 + 88 + 128 + 71 = 48,638
5. 319 + 72 + 507 + 22 + 918 + 52,079 + 14,200 + 366 + 52 + 368 + 45 = 68,948
6. 103 + 45 + 678 + 299 + 25,233 + 15 + 7 + 98 + 355 + 1,905 + 7,266 + 1,247 = 37,251
7. 46 + 1,872 + 5,521 + 27 + 604,711 + 198 + 12 + 91,722 + 434 = 704,543
8. 107,785 + 42 + 399 + 101 + 55 + 8 + 249 + 13 + 25,287 + 66 = 134,005
9. 18 + 275 + 39 + 44,209 + 1,552 + 8 + 91 + 241 + 918 + 77 = 47,428
10. 367 + 29 + 31 + 81,207 + 114,728 + 2,499 + 15 + 287 + 315 + 68 = 199,546

Exercise VIII:

1. 262 + 115 + 27,564 + 19 + 88 + 643 + 139 + 14 + 135 + 729 = 29,708
2. 47 + 126 + 318 + 9 + 35,903 + 817 + 62 + 18 + 52,907 + 38 + 249 = 90,494
3. 16 + 9 + 427 + 81 + 307 + 29,647 + 443 + 189 + 7,652 + 1,998 + 4,621 = 45,390
4. 1,515 + 2,706 + 45 + 218 + 976 + 312 + 22 + 92,877 + 505 + 12 = 99,188
5. 1,862 + 479 + 321 + 57 + 28,916 + 3,007 + 25 + 188 + 97 + 202 + 255,809 = 290,963
6. 3,019 + 52 + 257 + 1,972 + 886 + 50,298 + 681 + 72 + 8 + 118 + 22,813 + 18 = 80,194
7. 8,427 + 62 + 137 + 596 + 4 + 19,978 + 87 + 3 + 781 + 832 = 30,907
8. 94 + 19,853 + 758 + 6 + 3,555 + 3,872 + 49 + 6,172 + 153 + 840 + 8,129 = 43,481
9. 31,769 + 24 + 358 + 9 + 1,736 + 412 + 365 + 1,735 + 846 + 28 + 75 + 4 = 37,361
10. 1,027 + 46 + 329 + 115 + 13 + 7 + 107 + 8,909 + 16,722 + 382 + 149 = 27,806

Exercise IX:

1. 123 added to itself 9 times equals 1,107
2. 1,234 added to itself 9 times equals 11,106
3. 12,345 added to itself 9 times equals 111,105
4. 123,456 added to itself 9 times equals 1,111,104
5. 1,234,567 added to itself 9 times equals 11,111,103
6. 12,345,678 added to itself 9 times equals 111,111,102
7. 123,456,789 added to itself 9 times equals 1,111,111,101

Exercise X:

1. 1 + 2 + 3 + 4 + 5 + ... 10 = 55
2. 1 + 2 + 3 + 4 + 5 + ... 20 = 210
3. 1 + 2 + 3 + 4 + 5 + ... 30 = 465
4. 1 + 2 + 3 + 4 + 5 + ... 40 = 820
5. 1 + 2 + 3 + 4 + 5 + ... 50 = 1,275
6. 1 + 2 + 3 + 4 + 5 + ... 60 = 1,830
7. 1 + 2 + 3 + 4 + 5 + ... 70 = 2,485
8. 1 + 2 + 3 + 4 + 5 + ... 80 = 3,240
9. 1 + 2 + 3 + 4 + 5 + ... 90 = 4,095
10. 1 + 2 + 3 + 4 + 5 + ... 100 = 5,050

SUBTRACTION

Exercise I:

1. 61 - 8 - 6 - 2 - 1 - 7 - 9 - 4 - 9 = 15
2. 52 - 7 - 8 - 9 - 1 - 6 - 5 - 3 - 2 - 4 = 7
3. 50 - 2 - 5 - 7 - 8 - 6 - 9 - 3 - 5 = 5
4. 55 - 3 - 8 - 7 - 5 - 9 - 4 - 6 - 3 = 10
5. 49 - 5 - 1 - 7 - 3 - 5 - 8 - 4 - 6 - 2 = 8
6. 44 - 6 - 8 - 1 - 4 - 6 - 5 - 3 = 11
7. 54 - 7 - 2 - 9 - 4 - 1 - 3 - 9 - 5 = 14
8. 42 - 9 - 1 - 7 - 3 - 6 - 4 - 1 - 2 - 3 = 6
9. 48 - 8 - 5 - 3 - 4 - 2 - 9 - 6 - 7 = 4
10. 55 - 7 - 8 - 2 - 3 - 4 - 5 - 9 - 9 - 6 = 2

Exercise II:

1. 51 - 2 - 3 - 5 - 8 - 2 - 4 - 6 - 5 - 9 = 7
2. 38 - 2 - 4 - 1 - 5 - 4 - 3 - 2 - 6 - 5 = 6
3. 44 - 5 - 5 - 4 - 6 - 1 - 6 - 2 - 5 - 4 = 6
4. 50 - 3 - 6 - 5 - 3 - 8 - 1 - 9 - 6 = 9
5. 41 - 1 - 6 - 4 - 3 - 6 - 2 - 1 - 8 - 5 = 5
6. 104 - 6 - 3 - 1 - 4 - 8 - 2 - 5 - 3 - 9 - 7 - 4 - 8 - 5 - 1 - 9 - 7 - 3 - 9 = 10
7. 123 - 5 - 3 - 4 - 8 - 3 - 6 - 2 - 9 - 7 - 5 - 4 - 1 - 6 - 8 - 4 - 6 - 3 - 8 - 9 - 6 - 3 - 6 = 7
8. 104 - 9 - 1 - 2 - 7 - 8 - 9 - 3 - 5 - 9 - 1 - 8 - 6 - 7 - 3 - 1 - 4 - 2 - 5 = 14
9. 106 - 7 - 8 - 3 - 1 - 5 - 7 - 2 - 4 - 6 - 8 - 5 - 9 - 7 - 4 - 2 - 8 - 8 = 12
10. 104 - 5 - 7 - 9 - 8 - 3 - 6 - 6 - 8 - 3 - 1 - 1 - 4 - 4 - 5 - 9 - 7 = 18

Exercise III:

1. 614 - 75 - 81 - 27 - 30 - 45 - 98 - 94 - 62 - 83 = 19
2. 522 - 86 - 53 - 32 - 41 - 24 - 96 - 67 - 73 - 39 = 11
3. 526 - 53 - 34 - 25 - 76 - 39 - 64 - 73 - 59 - 81 = 22
4. 495 - 29 - 81 - 40 - 38 - 15 - 72 - 61 - 94 - 27 = 38
5. 667 - 72 - 16 - 55 - 91 - 88 - 27 - 83 - 72 - 69 = 94
6. 382 - 13 - 36 - 42 - 65 - 25 - 55 - 46 - 12 - 34 = 54
7. 389 - 12 - 62 - 13 - 42 - 34 - 63 - 25 - 60 - 32 = 46
8. 380 - 21 - 13 - 43 - 35 - 62 - 24 - 46 - 60 - 32 = 44
9. 520 - 31 - 24 - 95 - 33 - 59 - 66 - 62 - 85 - 17 = 48
10. 555 - 78 - 52 - 43 - 26 - 63 - 58 - 37 - 85 - 19 = 94

Exercise IV:

1. 368 - 16 - 33 - 14 - 20 - 98 - 57 - 46 - 27 - 19 = 38
2. 513 - 45 - 22 - 91 - 28 - 37 - 56 - 60 - 11 - 88 = 75
3. 567 - 42 - 18 - 76 - 99 - 53 - 75 - 32 - 81 - 22 = 69
4. 662 - 87 - 78 - 99 - 41 - 96 - 75 - 13 - 62 - 27 = 84
5. 614 - 19 - 83 - 62 - 94 - 98 - 45 - 30 - 27 - 81 = 75
6. 522 - 39 - 11 - 73 - 67 - 96 - 24 - 41 - 32 - 53 = 86
7. 526 - 22 - 81 - 59 - 73 - 64 - 39 - 76 - 25 - 34 = 53
8. 525 - 14 - 83 - 18 - 48 - 15 - 99 - 77 - 31 - 24 - 50 = 66
9. 491 - 16 - 47 - 55 - 19 - 22 - 96 - 33 - 87 - 45 = 71
10. 495 - 29 - 81 - 40 - 38 - 15 - 72 - 61 - 94 - 27 = 38

Exercise V:

1. 4,628 - 256 - 415 - 615 - 337 - 991 - 418 - 755 - 364 - 299 = 178
2. 5,568 - 332 - 987 - 772 - 456 - 345 - 123 - 789 - 456 - 321 = 987
3. 4,698 - 128 - 919 - 317 - 432 - 239 - 336 - 455 - 982 - 274 = 616
4. 5,950 - 619 - 111 - 418 - 547 - 995 - 723 - 418 - 413 - 789 = 917
5. 5,459 - 376 - 187 - 452 - 369 - 532 - 456 - 761 - 907 - 883 = 536
6. 5,078 - 318 - 999 - 782 - 645 - 321 - 118 - 227 - 389 - 645 = 634
7. 5,862 - 230 - 645 - 937 - 628 - 757 - 923 - 118 - 523 - 697 = 404
8. 6,425 - 616 - 782 - 654 - 329 - 917 - 818 - 217 - 324 - 916 = 852
9. 5,421 - 777 - 189 - 652 - 456 - 367 - 219 - 552 - 346 - 988 = 875
10. 5,791 - 377 - 462 - 555 - 380 - 919 - 827 - 345 - 628 - 872 = 426

Exercise VI:

1. 39,168 - 2,133 - 1,182 - 7,516 - 7,155 - 3,321 - 1,987 - 1,782 - 6,191 - 5,555 = 2,346
2. 56,445 - 2,336 - 7,529 - 7,718 - 8,177 - 6,152 - 6,881 - 7,521 - 1,111 - 3,378 = 5,642
3. 51,895 - 9,433 - 2,876 - 1,152 - 3,781 - 3,922 - 2,581 - 7,753 - 6,218 - 6,187 = 7,992
4. 44,498 - 1,787 - 8,724 - 3,335 - 1,785 - 9,192 - 3,326 - 5,578 - 2,741 - 6,832 = 1,198
5. 51,195 - 7,544 - 5,277 - 4,774 - 6,228 - 5,157 - 3,138 - 4,572 - 9,196 - 3,187 = 2,122
6. 40,312 - 1,876 - 1,971 - 2,789 - 6,656 - 1,198 - 7,233 - 2,918 - 3,641 - 8,655 = 3,375
7. 44,768 - 6,672 - 5,786 - 1,056 - 2,391 - 2,572 - 7,476 - 1,127 - 2,136 - 6,098 = 9,454
8. 54,387 - 6,787 - 9,211 - 3,618 - 7,774 - 6,253 - 1,877 - 8,875 - 1,962 - 3,312 = 4,718
9. 51,037 - 3,726 - 2,225 - 9,633 - 1,187 - 3,256 - 6,687 - 9,281 - 6,665 - 5,050 = 3,327
10. 45,252 - 3,347 - 4,654 - 2,831 - 1,845 - 8,369 - 3,594 - 4,854 - 1,041 - 5,860 = 8,857

Exercise VII:

1. 199,546 - 367 - 29 - 31 - 81,207 - 114,728 - 2,499 - 15 - 287 - 315 = 68
2. 47,428 - 77 - 918 - 241 - 91 - 8 - 1,552 - 18 - 44,209 - 39 = 275
3. 134,005 - 66 - 107,785 - 25,287 - 13 - 42 - 399 - 101 - 55 - 8 = 249
4. 704,972 - 434 - 46 - 1,872 - 91,722 - 12 - 604,711 - 27 - 5,521 - 429 = 198
5. 37,251 - 1,247 - 7,266 - 1,905 - 355 - 98 - 7 - 15 - 25,233 - 299 - 678 - 45 = 103
6. 276,294 - 187,622 - 29,500 - 31 - 457 - 218 - 99 - 10 - 57,298 - 44 - 133 = 882
7. 15,595 - 75 - 191 - 25 - 417 - 98 - 6,442 - 77 - 3,612 - 18 - 9 - 4,298 = 333
8. 176,111 - 1,032 - 3,519 - 17 - 42 - 379 - 25 - 25,261 - 35 - 144,729 = 1,072
9. 48,638 - 71 - 128 - 21 - 468 - 379 - 88 - 1,919 - 406 - 3 - 44,298 - 15 = 842
10. 68,948 - 45 - 368 - 52 - 366 - 14,200 - 52,079 - 918 - 22 - 507 - 72 = 319

Exercise VIII:

1. 29,708 - 262 - 115 - 27,564 - 19 - 88 - 643 - 139 - 14 - 135 = 729
2. 90,494 - 47 - 126 - 318 - 9 - 35,903 - 817 - 62 - 18 - 38 - 52,907 = 249
3. 45,390 - 4,621 - 1,998 - 16 - 9 - 7,652 - 189 - 443 - 307 - 29,647 - 81 = 427
4. 99,188 - 1,515 - 2,706 - 45 - 218 - 976 - 312 - 22 - 92,877 - 12 = 505
5. 290,963 - 255,809 - 202 - 97 - 188 - 25 - 3,007 - 28,916 - 57 - 321 - 479 = 1,862
6. 80,194 - 18 - 22,813 - 118 - 8 - 72 - 681 - 50,298 - 886 - 1,972 - 257 - 52 = 3,019
7. 30,907 - 8,427 - 62 - 137 - 596 - 4 - 19,978 - 87 - 3 - 781 = 832
8. 43,481 - 94 - 19,853 - 758 - 6 - 3,555 - 3,872 - 49 - 6,172 - 153 - 840 = 8,129
9. 37,361 - 4 - 75 - 28 - 31,769 - 24 - 358 - 9 - 1,736 - 412 - 365 - 1,735 = 846
10. 27,806 - 1,027 - 46 - 329 - 115 - 13 - 7 - 107 - 8,909 - 149 - 16,722 = 382

Exercise IX:

1. 1,107 - 123 nine times = 0
2. 11,106 - 1,234 nine times = 0

3. 111,105 - 12,345 nine times = 0
4. 1,111,104 - 123,456 nine times = 0
5. 11,111,103 - 1,234,567 nine times = 0
6. 111,111,102 - 12,345,678 nine times = 0
7. 1,111,111,101 - 123,456,789 nine times = 0

MIXED NUMBERS

Addition and Subtraction

Exercise I:

1. 382 + 119 + 75 + 13 + 28 + 753 - 812 - 49 + 72 + 311 + 496 = 1,388
2. 694 + 273 + 145 - 288 - 91 - 303 + 618 + 12 + 37 + 65 = 1,162
3. 432 + 79 + 816 + 55 - 44 - 219 + 81 - 38 - 645 + 17 = 534
4. 25 + 199 + 86 + 314 + 275 - 318 - 66 - 95 + 32 - 67 - 83 = 302
5. 718 + 362 - 95 - 88 + 79 + 811 + 46 - 459 - 247 - 36 = 1,091
6. 316 + 45 + 289 - 92 - 16 - 496 + 218 - 25 + 358 + 415 = 1,012
7. 18 + 145 + 72 + 33 - 98 + 317 - 246 - 19 - 82 + 3 + 412 = 555
8. 222 + 319 + 85 - 73 - 218 - 38 - 42 - 18 + 300 + 421 = 958
9. 505 + 277 + 918 + 365 + 24 - 289 - 39 - 64 - 52 - 3 + 41 = 1,683
10. 9 + 85 + 311 - 29 - 87 - 52 + 428 + 33 + 29 + 40 - 321 = 446

Exercise II:

1. 52 - 22 - 18 + 29 + 36 + 111 + 202 - 300 + 45 + 276 - 81 = 330
2. 96 - 25 + 14 - 38 + 251 + 312 - 488 + 12 - 39 - 78 + 62 = 79
3. 19 + 28 + 145 - 28 - 77 - 4 + 51 + 6 + 98 + 7 + 15 - 98 - 46 = 116
4. 85 + 246 + 31 + 52 - 76 - 21 - 19 + 47 + 36 + 112 + 309 - 18 = 784
5. 245 + 16 + 72 + 119 - 32 - 91 - 84 + 64 + 51 - 38 - 79 = 243
6. 436 + 118 + 29 - 85 - 201 - 59 - 4 + 56 - 16 + 219 - 77 = 416
7. 23 - 8 + 5 + 66 + 45 + 89 - 109 - 23 + 77 + 91 + 82 + 35 - 88 = 285
8. 808 + 379 + 645 - 819 - 72 - 64 - 30 + 29 + 75 + 36 + 111 = 1,098
9. 19 + 28 + 37 - 44 - 9 + 304 + 88 + 956 - 337 - 93 - 62 + 32 = 919
10. 614 + 71 + 22 - 18 - 46 - 6 + 67 + 152 + 48 - 84 - 21 + 14 = 813

Appendix 10

MULTIPLICATION TABLES

1 x 1 = 1	2 x 1 = 2	3 x 1 = 3	4 x 1 = 4
1 x 2 = 2	2 x 2 = 4	3 x 2 = 6	4 x 2 = 8
1 x 3 = 3	2 x 3 = 6	3 x 3 = 9	4 x 3 = 12
1 x 4 = 4	2 x 4 = 8	3 x 4 = 12	4 x 4 = 16
1 x 5 = 5	2 x 5 = 10	3 x 5 = 15	4 x 5 = 20
1 x 6 = 6	2 x 6 = 12	3 x 6 = 18	4 x 6 = 24
1 x 7 = 7	2 x 7 = 14	3 x 7 = 21	4 x 7 = 28
1 x 8 = 8	2 x 8 = 16	3 x 8 = 24	4 x 8 = 32
1 x 9 = 9	2 x 9 = 18	3 x 9 = 27	4 x 9 = 36
5 x 1 = 5	6 x 1 = 6	7 x 1 = 7	8 x 1 = 8
5 x 2 = 10	6 x 2 = 12	7 x 2 = 14	8 x 2 = 16
5 x 3 = 15	6 x 3 = 18	7 x 3 = 21	8 x 3 = 24
5 x 4 = 20	6 x 4 = 24	7 x 4 = 28	8 x 4 = 32
5 x 5 = 25	6 x 5 = 30	7 x 5 = 35	8 x 5 = 40
5 x 6 = 30	6 x 6 = 36	7 x 6 = 42	8 x 6 = 48
5 x 7 = 35	6 x 7 = 42	7 x 7 = 49	8 x 7 = 56
5 x 8 = 40	6 x 8 = 48	7 x 8 = 56	8 x 8 = 64
5 x 9 = 45	6 x 9 = 54	7 x 9 = 63	8 x 9 = 72
9 x 1 = 9			
9 x 2 = 18			
9 x 3 = 27			
9 x 4 = 36			
9 x 5 = 45			
9 x 6 = 54			
9 x 7 = 63			
9 x 8 = 72			
9 x 9 = 81			

MULTIPLICATION

Exercise I:

1. 8 x 21,212 = 169,696
2. 4 x 63,240 = 252,960
3. 5 x 200 = 1,000
4. 8 x 76,455 = 611,640
5. 3 x 45,678 = 137,034
6. 6 x 80,075 = 480,450
7. 4 x 86,443 = 345,772
8. 4 x 364,857 = 1,459,428
9. 3 x 22,336 = 67,008
10. 5 x 37,889 = 189,445

Exercise II:

1. 8 x 48,652 = 389,216
2. 8 x 277,653 = 2,221,224
3. 7 x 9,764,318 = 68,350,226
4. 6 x 23,047 = 138,282
5. 3 x 402,607 = 1,207,821
6. 7 x 283,462 = 1,984,234
7. 8 x 1,217 = 9,736
8. 2 x 30,062 = 60,124
9. 9 x 7,007 = 63,063
10. 8 x 425 = 3,400

Exercise III:

1. 45 x 2,015 = 90,675
2. 35 x 3,261 = 114,135
3. 123 x 123 = 15,129
4. 327 x 43 = 14,061
5. 654 x 57 = 37,278
6. 46 x 8,107 = 372,922
7. 123 x 456 = 56,088
8. 375 x 4,877 = 1,828,875
9. 27 x 12,345 = 333,315
10. 306 x 7,004 = 2,143,224

Exercise IV:

1. 456 x 654 = 298,224
2. 847 x 1,173 = 993,531
3. 231 x 426 = 98,406

4. 408 x 3,371 = 1,375,368
5. 199 x 6,547 = 1,302,853
6. 289 x 3,476 = 1,004,564
7. 423 x 6,115 = 2,586,645
8. 196 x 587 = 115,052
9. 296 x 5,899 = 1,746,104
10. 874 x 1,107 = 967,518

Exercise V:

1. 722 x 351 = 253,422
2. 414 x 616 = 255,024
3. 718 x 622 = 446,596
4. 303 x 652 = 197,556
5. 215 x 346 = 74,390
6. 524 x 719 = 376,756
7. 112 x 103 = 11,536
8. 775 x 623 = 482,825
9. 418 x 769 = 321,442
10. 216 x 107 = 23,112

Exercise VI:

1. 446 x 778 = 346,988
2. 139 x 874 = 121,486
3. 986 x 143 = 140,998
4. 147 x 600 = 88,200
5. 205 x 502 = 102,910

Exercise VII:

1. 9 x 123,456,789 = 1,111,111,101
2. 18 x 123,456,789 = 2,222,222,202
3. 27 x 123,456,789 = 3,333,333,303
4. 36 x 123,456,789 = 4,444,444,404
5. 45 x 123,456,789 = 5,555,555,505
6. 56 x 123,456,789 = 6,666,666,606
7. 63 x 123,456,789 = 7,777,777,707
8. 72 x 123,456,789 = 8,888,888,808
9. 81 x 123,456,789 = 9,999,999,909

DIVISION

Exercise I:

1. 366 ÷ 6 = 61
2. 816 ÷ 8 = 102
3. 609 ÷ 3 = 203
4. 654 ÷ 2 = 327
5. 72 ÷ 4 = 18
6. 125 ÷ 5 = 25
7. 91 ÷ 7 = 13
8. 216 ÷ 6 = 36
9. 632 ÷ 8 = 79
10. 264 ÷ 3 = 88

Exercise II:

1. 110 ÷ 5 = 22
2. 360 ÷ 8 = 45
3. 387 ÷ 9 = 43
4. 588 ÷ 6 = 98
5. 2,172 ÷ 3 = 724
6. 3,400 ÷ 8 = 425
7. 1,022 ÷ 7 = 146
8. 654 ÷ 2 = 327
9. 1,985 ÷ 5 = 397
10. 738 ÷ 6 = 123

Exercise III:

1. 138,282 ÷ 6 = 23,047
2. 22,212 ÷ 9 = 2,468
3. 387,628 ÷ 4 = 96,907
4. 1,207,821 ÷ 3 = 402,607
5. 1,111,111,101 ÷ 9 = 123,456,789

Exercise IV:

1. 2,143,224 ÷ 306 = 7,004
2. 298,224 ÷ 456 = 654
3. 591,682 ÷ 91 = 6,502
4. 408 ÷ 34 = 12
5. 1,035 ÷ 45 = 23
6. 1,904 ÷ 56 = 34
7. 1,296 ÷ 72 = 18
8. 3,196 ÷ 68 = 47

9. 14,061 ÷ 43 = 327
10. 37,278 ÷ 57 = 654

Exercise V:

1. 372,922 ÷ 8,107 = 46
2. 14,868 ÷ 236 = 63
3. 56,088 ÷ 456 = 123
4. 9,231 ÷ 543 = 17
5. 9,821 ÷ 161 = 61
6. 21,472 ÷ 671 = 32
7. 14,202 ÷ 789 = 18
8. 42,048 ÷ 876 = 48
9. 298,224 ÷ 654 = 456
10. 591,682 ÷ 6,502 = 91

Exercise VI:

1. 546 ÷ 21 = 26
2. 774 ÷ 18 = 43
3. 9,744 ÷ 112 = 87
4. 8,946 ÷ 142 = 63
5. 90,675 ÷ 2,015 = 45
6. 7,964 ÷ 22 = 362
7. 5,616 ÷ 24 = 234
8. 993,531 ÷ 1,173 = 847
9. 98,406 ÷ 231 = 426
10. 114,135 ÷ 3,261 = 35